Sledgehammering Without ATPs

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Proof AutomationTools in Isabelle/HOL

Proof method

Try to automatically solve a proof goal using different algorithm or heuristics.

 $\textbf{Examples} \text{ simp, auto, blast, metis,} \dots$

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Push-button automation using internal proof methods.

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Push-button automation using **internal proof methods**.

Sledgehammer

Push-button automation using external automatic theorem provers (ATPs).

Components Relevance filter, translation module, external ATPs, proof minimization, proof reconstruction

Proof Automation User Perspective

Automation-driven development

- 1. State the proof goal.
- 2. Try automation (e.g., try0 or Sledgehammer).
- 3. Stop if proof found; otherwise, find an intermediate lemma to help automation and go to step 1.

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Caveats:

- try0 and Sledgehammer have different strengths: Manually call both.
- try0 has no fact filter: Manually provide facts.

ATP-Free Hammer

We propose an alternative, ATP-free hammer:

- 1. Relevance filter: as before
- 2. Translation module: not necessary
- 3. External ATPs: replaced by internal proof methods
- 4. Proof minimization: as before
- 5. Proof reconstruction: not necessary

Motivating Example

```
\begin{array}{l} \textbf{have}\ \mathsf{map2}\ (+)\ (\mathsf{map2}\ (+)\ \mathit{xs}\ \mathit{ys})\ \mathit{zs} = \mathsf{map2}\ (+)\ \mathit{xs}\ (\mathsf{map2}\ (+)\ \mathit{ys}\ \mathit{zs}) \\ \textbf{for}\ \mathit{xs}\ \mathit{ys}\ \mathit{zs} :: \ (\text{`a}\ ::\ \mathsf{ab\_semigroup\_add})\ \mathsf{list} \end{array}
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Automation-driven development:

- 1. try0: Timeout, no proof found.
- 2. ATP hammer: Timeout, no proof found.
- 3. ATP-free hammer: Proof found!

by (simp add: case_prodI2 prod.case_distrib zip_assoc case_prod_app map_zip_map map_zip_map2 ab_semigroup_add_class.add_ac(1))

Experiment Methodology

Benchmark

We randomly selected 20 formalizations and, in each, 250 goals: a total of 5000 tests cases.

Experiment Methodology

Benchmark

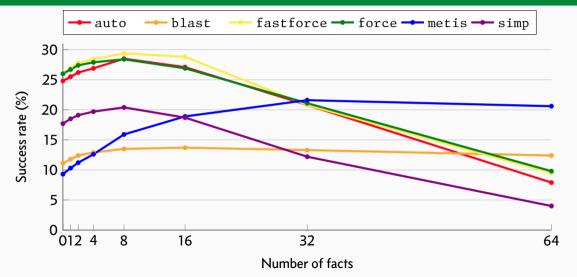
We randomly selected 20 formalizations and, in each, 250 goals: a total of 5000 tests cases.

Configurations

We tested 13 proof methods with a 2-second timeout and between 0 and 64 facts.

The 13 proof methods with 0 facts correspond to try0.

ExperimentDetailed Results (Selected Proof Methods)



ExperimentOverall Results

Configurations	Success rate (%)
try0	28.5
ATP hammer	72.1
ATP-free hammer	46.8
ATP hammer \cup try 0	74.1
ATP hammer \cup ATP-free hammer	74.6

Remainder: $try0 \subseteq ATP$ -free hammer

Conclusion: try0 and ATP-free hammer both improve the ATP hammer!

Discussion and Conclusion

We implemented and evaluated an **ATP-free hammer** in Isabelle/HOL. It can find **new proofs** that the ATP hammer cannot.

We extended Sledgehammer with try0 and ATP-free portfolio.

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Take-home message

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- No need to manually call try0
- Expect better results

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Thank you!