Mechanizing the Splitting Framework

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ITP

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Motivation

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Is AVATAR refutational complete?

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Yes!*

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Spoiler

Yes!*

* under some fairness conditions, as described in our framework [E. et. al. 2021] that is kind of tricky so we decided to verify it with the proof assistant Isabelle/HOL but there is a lot of work to do so for now we are only done with a simple version of splitting and this is what this talk is about.

Splitting Framework

Saturation

saturate (verb): apply a given calculus and given redundancy deletion techniques to a set of formulas up to the limit.

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With resolution and subsumption

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Desired Properties for a Saturation-based Calculus [B. & G. 2001, W. et al. 2020]

Soundness

inferences: $\dfrac{P_1 \qquad \ldots \qquad P_n}{C}$ implies $\{P_1, \ldots, P_n\} \models \{C\}$

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$N_0 \models \{\bot\}$

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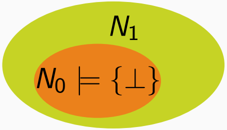

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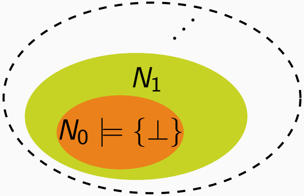

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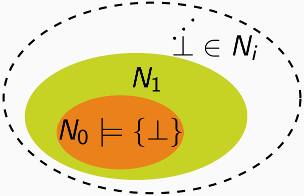

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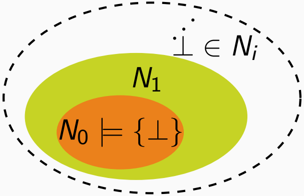

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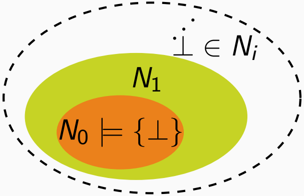

Static Completeness

N saturated and $N \models \{\bot\}$ implies $\bot \in N$.

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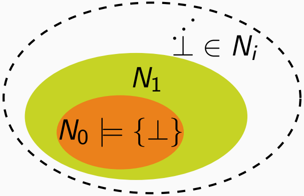

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Splitting

Intuition

Partition the search space by splitting a formula into independent subformulas.

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With resolution

$$\begin{array}{c} \neg P(a) \qquad P(x) \vee Q(y,b) \qquad \neg Q(z,z) \\ \hline\hline | \end{array}$$

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Partition the search space by splitting a formula into independent subformulas.

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$$\begin{array}{c} \neg P(a) \qquad P(x) \lor Q(y, b) \qquad \neg Q(z, z) \\ \hline\hline \dfrac{\neg P(a) \qquad P(x)}{\bot} \qquad \neg Q(z, z) \qquad \mid \end{array}$$

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Works for any saturation-based calculus!

Properties Specific to Splitting

Static Completeness

If

- base calculus statically complete
- N saturated
- $N \models \{\bot\}$

then $\bot \in N$.

Dynamic Completeness

If

- base calculus dynamically complete
- derivation $(N_i)_i$ *fair*
- $N_0 \models \{\bot\}$

then $\bot \in N_i$ for some i.

Properties Specific to Splitting

Strong Static Completeness

If

- base calculus statically complete
- N locally saturated
- $N \models \{\bot\}$

then $\bot \in N$.

Strong Dynamic Completeness

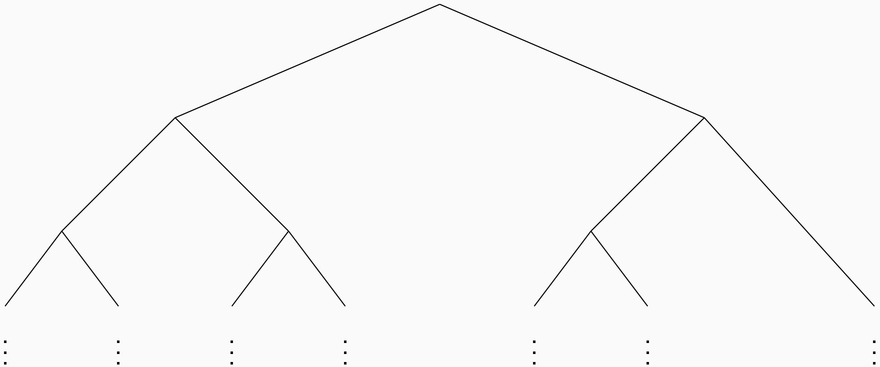
If

- base calculus dynamically complete
- derivation $(N_i)_i$ locally *fair*
- $N_0 \models \{\bot\}$

then $\bot \in N_i$ for some i.

Locality

Saturation and fairness are too strong.



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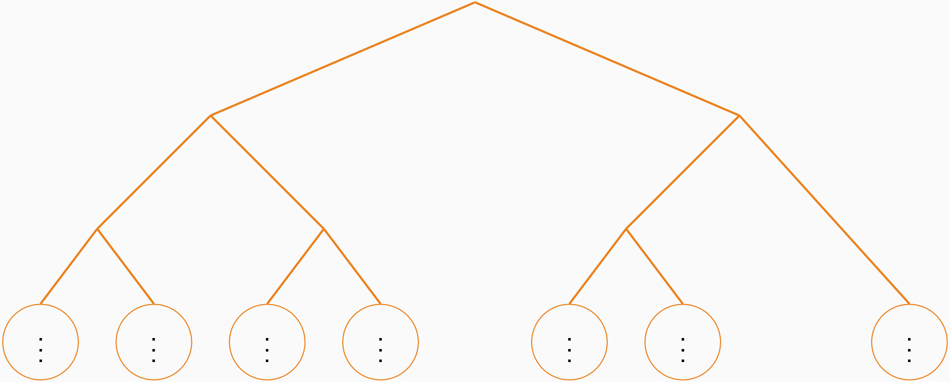

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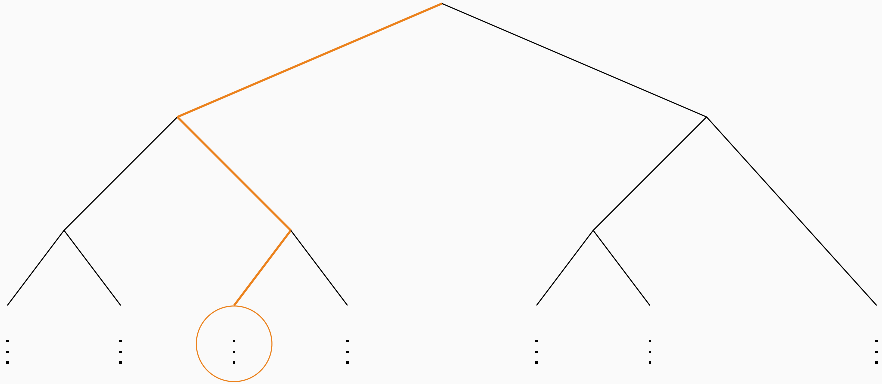

Mechanizing Splitting

Rules, Soundness and Completeness

Rules: Base, Unsat (mandatory)

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Soundness:

	✎	Isabelle		✎	Isabelle
Base	✓	✓	StrongUnsat	✓	✓
Unsat	✓	✓	Split	✗	✓
Tauto	✓	✓	Trim	✗	✓
Approx	✓	✓	Collect	✗	✓

Table 1: Inferences (premises $\models$ conclusions)

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Rules: Base, Unsat (mandatory) Tauto, Approx, StrongUnsat (optional)

Simplifications: Split, Trim, Collect (optional)

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Table 1: Inferences (premises $\models$ conclusions)

	📝	Isabelle
Split	✗	✓
Trim	✗	✓
Collect	✓	✓

Table 2: Simplifications (conclusions $\models$ premises)

Rules, Soundness and Completeness

Rules: BASE, UNSAT (mandatory)   TAUTO, APPROX, STRONGUNSAT (optional)

Simplifications: SPLIT, TRIM, COLLECT (optional)

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SPLIT	✗	✓
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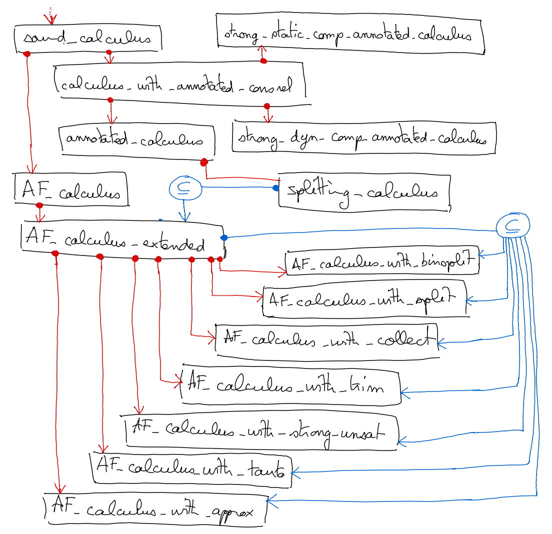
Completeness:

static ✓   dynamic ✓   strong static ✓   strong dynamic ✓

Modularity

Locale Structure (excerpt):



Modeling Lightweight AVATAR with Resolution

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The model

- uses binary-only splitting BinSplit (sound ✓) instead of Split;

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- discharges assumptions.

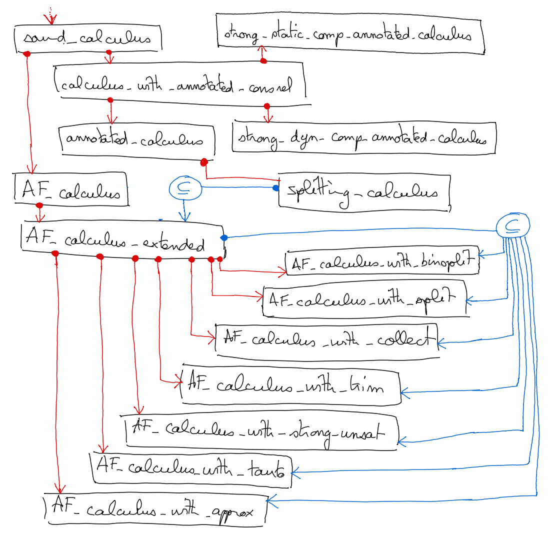
Resolution in Isabelle/HOL

Three entries in the Archive of Formal Proofs:

- **The Resolution Calculus for First-Order Logic**
  Anders Schlichtkrull, 2016.
- **Formalization of Bachmair and Ganzinger's Ordered Resolution Prover**
  Anders Schlichtkrull, Jasmin Blanchette, Dmitriy Traytel, Uwe Waldmann, 2018
- **Extensions to the Comprehensive Framework for Saturation Theorem Proving**
  Jasmin Blanchette, Sophie Tourret, 2020.

Plug and Play

Locale Structure (excerpt):



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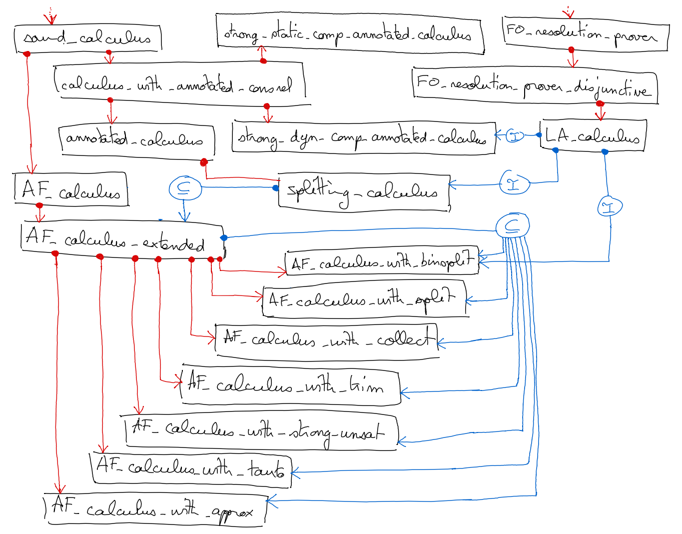
Locale Structure (excerpt) + Resolution:



Conjunctive vs Disjunctive Entailment

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- $\{\bot\} \models N$
- subsets entailed (non-strict)
- $(\forall C \in N_2.\ N_1 \models \{C\}) \implies N_1 \models N_2$
- transitivity

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- supersets entailment (both sides)
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$\models_\wedge$ can be defined easily from $\models_\vee$ and agree on atomic sets.

Defining a Suitable Disjunctive Entailment from a Conjunctive One

$$M \models_{\vee} N \quad = \quad M \models_{\wedge} N$$

- ✗ $\{\bot\} \models \{\}$
- ● reflexivity
- ✗ supersets entailment (both sides)
- ✗ cut elimination
- ✗ compactness (a form of)

Defining a Suitable Disjunctive Entailment from a Conjunctive One

$$\begin{aligned} M \models_{\vee} N &= \cancel{M \models_{\wedge} N} \\ &= \exists C \in N.\ M \models_{\wedge} \{C\} \end{aligned}$$

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- ✗ compactness (a form of)

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✗ compactness (a form of) unless $\models_\wedge$ is already compact

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- $\{\bot\} \models \{\}$
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- supersets entailment (both sides)
- cut elimination
- compactness (a form of)

Work Done and Perspectives

Isabelle/HOL mechanization

Splitting Framework

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  - ✓ preliminary notions ($\sim$ 3500 lines)

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Splitting Framework

Thank you!

Splitting Rules

$$\frac{C}{\bot \leftarrow \{\neg a_i\}_{i=1}^{n} \qquad\qquad (C_i \leftarrow \{a_i\})_{i=1}^{n}}\ \textsc{Split} \qquad \text{if } C \text{ splittable into } C_1, \ldots, C_n$$

Splitting Rules

$$\frac{C \leftarrow A}{\bot \leftarrow \{\neg a_i\}_{i=1}^{n} \cup A \qquad (C_i \leftarrow \{a_i\})_{i=1}^{n}} \textsc{Split} \qquad \text{if } C \text{ splittable into } C_1, \ldots, C_n$$

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- approximate formula with constraint

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- approximate formula with constraint
- remove formula with unsat constrainst

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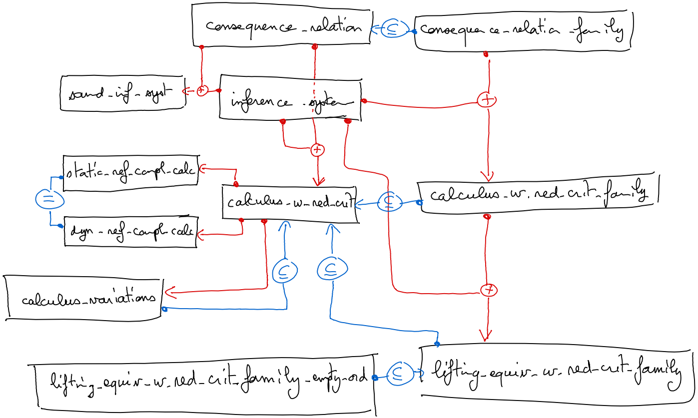
$+$ more optional rules

- approximate formula with constraint
- remove formula with unsat constrainst
- trim constraints
- ...

Finding a Suitable Redundancy Criterion

Saturation Framework's Core:



Finding a Suitable Redundancy Criterion

Saturation Framework's Core + Ordered Resolution: