Formalising Subject Reduction and Progress for Multiparty Session Processes

ITP 2025

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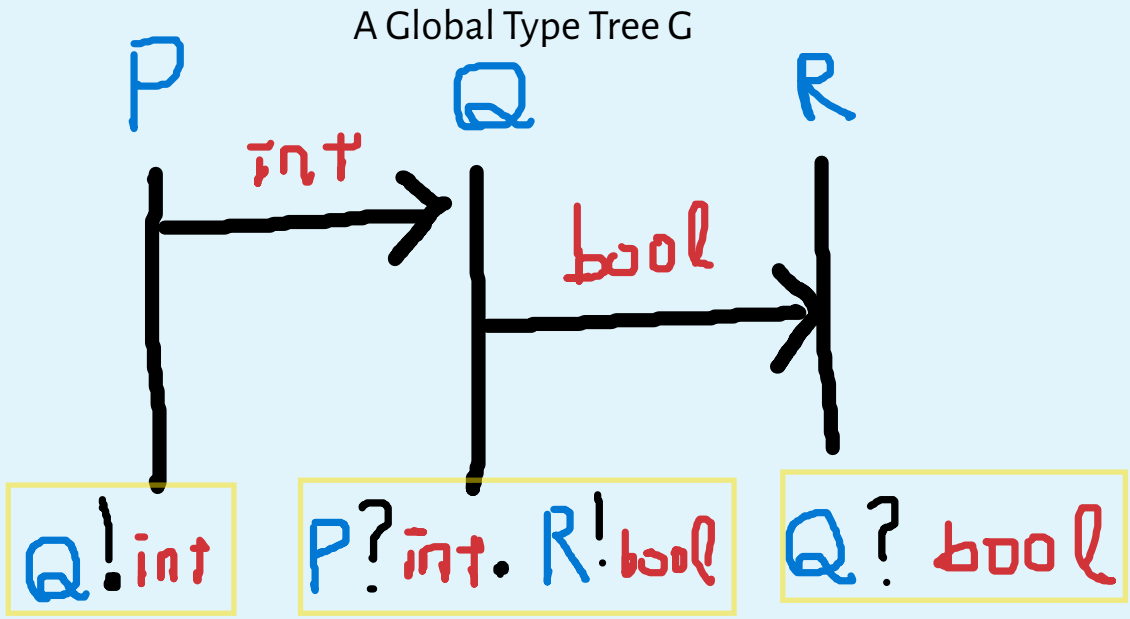
September 28, 2025

Outline

Multiparty Session Types (Top Down Approach) (Honda, NY and Carbone 2008)



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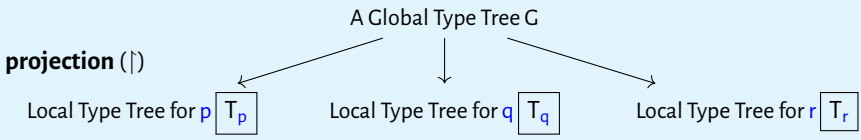

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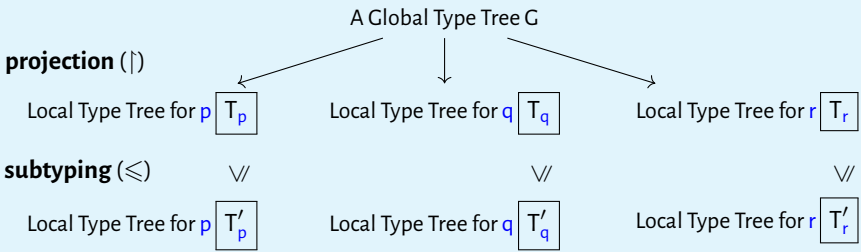

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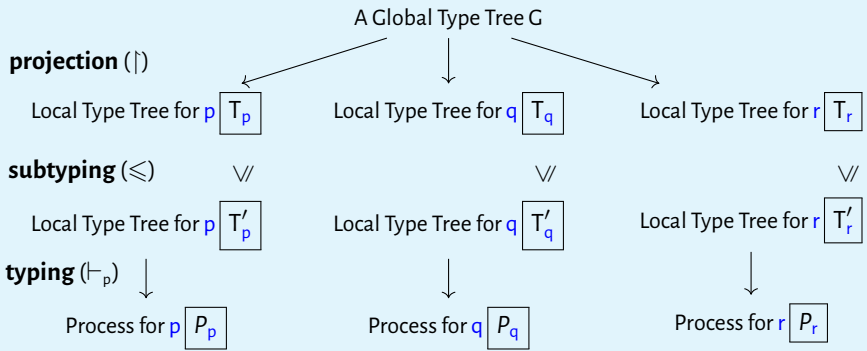

Contributions & Challenges

- extend synchronous MPST [5] with a **mechanised proof of the non-stuck theorem**

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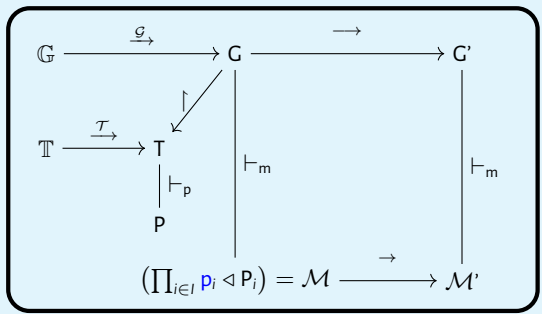
- decompose **balanced** type trees into finite prefixes – enabling inductive reasoning within infinite trees
- employ **finite lists** to encode, in Rcoq, continuations and branching/selections for type trees – simplifying coinductive definitions and proofs further

Design Overview



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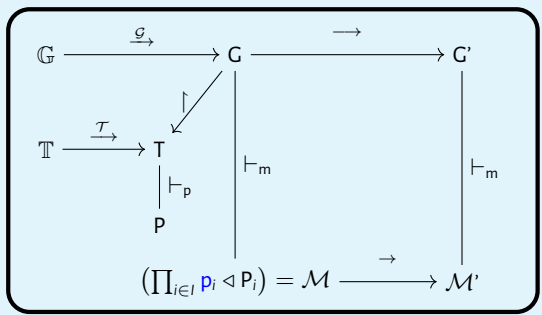
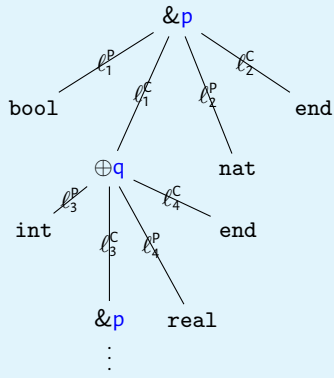


Outline

Session Trees (coinductive syntax)

$$\mathsf{T} \quad ::= \quad \mathtt{end} \quad \mid \quad \&_{i \in I} \mathsf{p}?\ell_i(\mathsf{S}_i).\mathsf{T}_i \quad \mid \quad \oplus_{i \in I} \mathsf{p}!\ell_i(\mathsf{S}_i).\mathsf{T}_i$$



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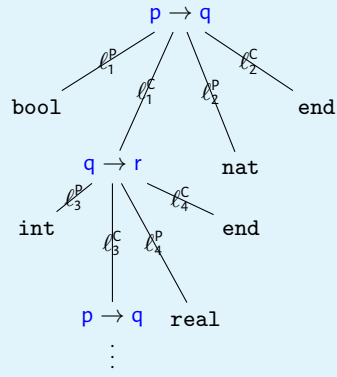

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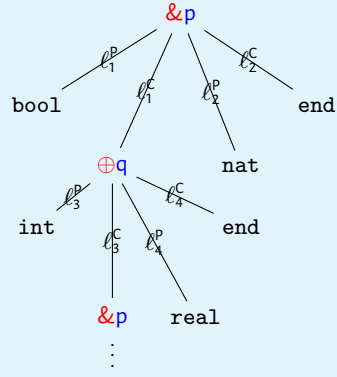

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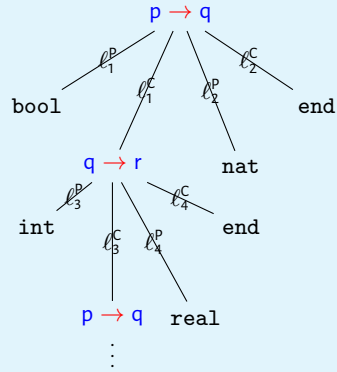

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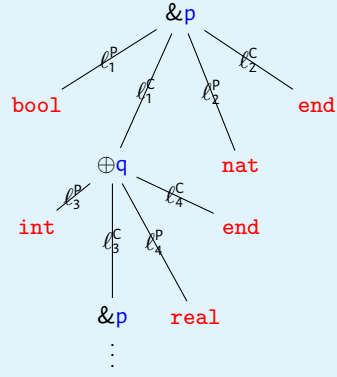

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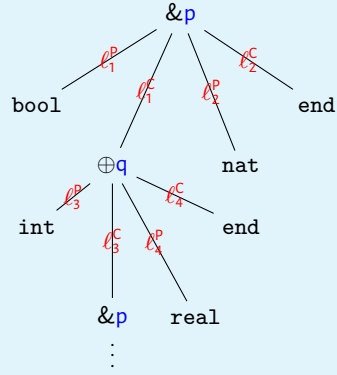

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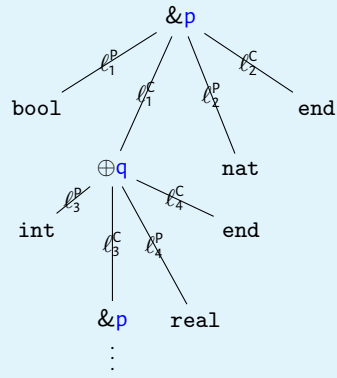

Global Types $\to$ Global Type Trees

represent recursive types and their unfoldings with the same tree – equi-recursive approach:

$$\frac{\mathbb{G}[\mu \mathbf{t}.\mathbb{G}/\mathbf{t}] \xrightarrow{\mathcal{G}} \mathsf{G}}{\mu \mathbf{t}.\mathbb{G} \xrightarrow{\mathcal{G}} \mathsf{G}}\ [\text{gtrans-rec}] \qquad \frac{\forall i \in I, \quad \mathbb{G}_i \xrightarrow{\mathcal{G}} \mathsf{G}_i}{\mathsf{p} \to \mathsf{q} : \{\ell_i(\mathsf{S}_i).\mathbb{G}_i\}_{i\in I} \xrightarrow{\mathcal{G}} \mathsf{p} \to \mathsf{q} : \{\ell_i(\mathsf{S}_i).\mathsf{G}_i\}_{i\in I}}\ [\text{gtrans-send}]$$

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Example

$$\mathbb{G} = \mu \mathbf{t}.\mathsf{p} \to \mathsf{q} \begin{cases} \ell_1(\mathtt{bool}).\mathbf{t} \\ \ell_2(\mathtt{nat}).\mathtt{end} \end{cases}$$

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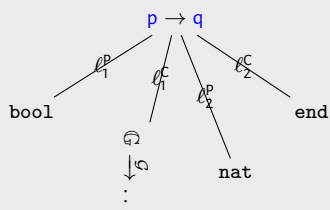

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Projection

extracting local type trees from global type trees:

$$\frac{\forall i \in I, G_i \upharpoonright_r T_i}{r \to q : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright_r \bigoplus_{i\in I} q!\ell_i(S_i).T_i} \text{ [proj-send]} \qquad \frac{\forall i \in I, G_i \upharpoonright_r T_i}{p \to r : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright_r \&_{i\in I}\, p?\ell_i(S_i).T_i} \text{ [proj-recv]}$$

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Provable


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Step Relation

global type trees evolve by consuming communications:

$$\frac{\forall i \in I \qquad \exists k \in I, \ell = \ell_k}{\left(\mathsf{p} \to \mathsf{q} : \{\ell_i(\mathsf{S}_i).\mathsf{G}_i\}_{i\in I}\right) \setminus \mathsf{p} \xrightarrow{\ell} \mathsf{q}\ \mathsf{G}_k} \text{[st-eq]}$$

$$\frac{\{\mathsf{r},\mathsf{s}\} \cap \{\mathsf{p},\mathsf{q}\} = \varnothing \qquad \forall i \in I, \{\mathsf{p},\mathsf{q}\} \subseteq \mathtt{pt}(\mathsf{G}_i)}{\left(\mathsf{r} \to \mathsf{s} : \{\ell_i(\mathsf{S}_i).\mathsf{G}_i\}_{i\in I}\right) \setminus \mathsf{p} \xrightarrow{\ell} \mathsf{q}\ \left(\mathsf{r} \to \mathsf{s} : \{\ell_i(\mathsf{S}_i).\mathsf{G}_i \setminus \mathsf{p} \xrightarrow{\ell} \mathsf{q}\}_{i\in I}\right)} \text{[st-neq]}$$

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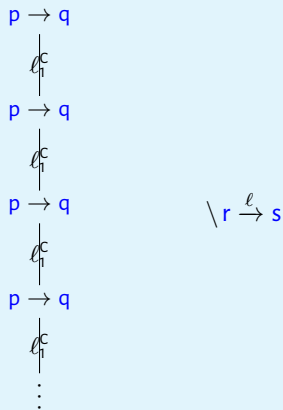
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Subtyping

provides flexibility to type system – use a process of type T whenever a process of type T′ is needed – $\mathsf{T} \leqslant \mathsf{T}'$

$$\frac{\forall i \in I, \quad \mathsf{S}_i \leq: \mathsf{S}'_i \quad \mathsf{T}_i \leqslant \mathsf{T}'_i}{\bigoplus_{i\in I} \mathsf{p}!\ell_i(\mathsf{S}_i).\mathsf{T}_i \leqslant \bigoplus_{i\in I\cup J} \mathsf{p}!\ell_i(\mathsf{S}'_i).\mathsf{T}'_i}\ [\text{sub-out}] \qquad \frac{\forall i \in I, \quad \mathsf{S}'_i \leq: \mathsf{S}_i \quad \mathsf{T}_i \leqslant \mathsf{T}'_i}{\&_{i\in I\cup J} \mathsf{p}?\ell_i(\mathsf{S}_i).\mathsf{T}_i \leqslant \&_{i\in I} \mathsf{p}?\ell_i(\mathsf{S}'_i).\mathsf{T}'_i}\ [\text{sub-in}]$$

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Type Checking

$$\frac{\forall i \in I, \quad \Gamma, x_i : \mathsf{S}_i \vdash_\mathsf{p} \mathsf{P}_i : \mathsf{T}_i}{\Gamma \vdash_\mathsf{p} \sum_{i\in I} \mathsf{p}?\ell_i(x_i).\mathsf{P}_i : \&_{i\in I} \mathsf{p}?\ell_i(\mathsf{S}_i).\mathsf{T}_i}\ [\text{tin}] \qquad \frac{\Gamma \vdash_\mathsf{s} e : \mathsf{S} \quad \Gamma \vdash_\mathsf{p} \mathsf{P} : \mathsf{T}}{\Gamma \vdash_\mathsf{p} \mathsf{p}!\ell(e).\mathsf{P} : \bigoplus \mathsf{p}!\ell(\mathsf{S}).\mathsf{T}}\ [\text{tout}]$$

$$\frac{\Gamma, \mathbf{X} : \mathsf{T} \vdash_\mathsf{p} \mathsf{P} : \mathsf{T}}{\Gamma \vdash_\mathsf{p} \mu\mathbf{X}.\mathsf{P} : \mathsf{T}}\ [\text{trec}] \qquad \frac{\Gamma \vdash_\mathsf{p} \mathsf{P} : \mathsf{T} \quad \mathsf{T} \leqslant \mathsf{T}'}{\Gamma \vdash_\mathsf{p} \mathsf{P} : \mathsf{T}'}\ [\text{tsub}] \qquad \frac{\Gamma \vdash_\mathsf{s} e : \texttt{bool} \quad \Gamma \vdash_\mathsf{p} \mathsf{P}_1 : \mathsf{T} \quad \Gamma \vdash_\mathsf{p} \mathsf{P}_2 : \mathsf{T}}{\Gamma \vdash_\mathsf{p} \text{if } e \text{ then } \mathsf{P}_1 \text{ else } \mathsf{P}_2 : \mathsf{T}}\ [\text{tite}]$$

$$\frac{\forall i \in I, \quad \mathsf{G}\upharpoonright_{\mathsf{p}_i} \mathsf{T}_i \quad \vdash_\mathsf{p} \mathsf{P}_i : \mathsf{T}_i \quad \mathtt{pt}(\mathsf{G}) \subseteq \{\mathsf{p}_i \mid i \in I\}}{\vdash_\mathsf{m} \prod_{i\in I} \mathsf{p}_i \lhd \mathsf{P}_i : \mathsf{G}}\ [\text{tsess}]$$

Processes and Multiparty Sessions (syntax)

$$\begin{array}{lcl} \mathtt{P} & ::= & \mathtt{p}!\ell(e).\mathtt{P} \mid \sum_{i\in I} \mathtt{p}?\ell_i(x_i).\mathtt{P}_i \mid \mathtt{if}\ \mathtt{e}\ \mathtt{then}\ \mathtt{P}\ \mathtt{else}\ \mathtt{P} \mid \mu\mathbf{X}.\mathtt{P} \mid \mathbf{X} \mid \mathbf{0} \\ \mathcal{M} & ::= & \mathtt{p} \triangleleft \mathtt{P} \mid \mathcal{M} \mid \mathcal{M} \end{array}$$

Processes and Multiparty Sessions (pre-congruence and reduction)

$$\frac{}{\mathtt{p} \triangleleft \mu\mathbf{X}.\mathtt{P} \mid \mathcal{M} \Rrightarrow \mathtt{p} \triangleleft \mathtt{P}[\mu\mathbf{X}.\mathtt{P}/\mathbf{X}] \mid \mathcal{M}}\ [\text{po-unf}]$$

$$\frac{\forall i \in I \quad j \in I \quad e \downarrow v}{\mathtt{p} \triangleleft \sum_{i\in I} \mathtt{q}?\ell_i(x_i).\mathtt{P}_i \mid \mathtt{q} \triangleleft \mathtt{p}!\ell_j(e).\mathtt{Q} \mid \mathcal{M} \longrightarrow \mathtt{p} \triangleleft \mathtt{P}_j[v/x_j] \mid \mathtt{q} \triangleleft \mathtt{Q} \mid \mathcal{M}}\ [\text{r-comm}]$$

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imported by some recently published work [13, 2]

Outline

Balancedness

G is balanced if, for every subtree G' of G, whenever $p \in \mathtt{pt}(G')$, $\exists\, k \in \mathbb{N}$ such that

1. For all paths γ of length k from the root of G', $p \in \gamma$
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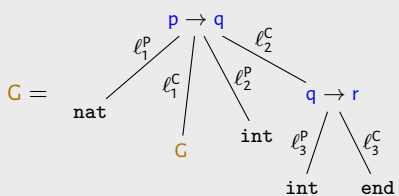
Example (an unbalanced tree)



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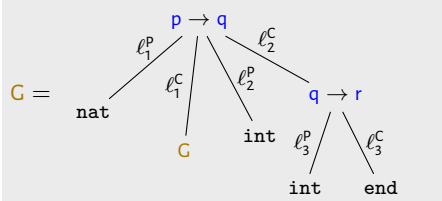

Well-formedness

Global type tree G is *well-formed* if $\exists$ global type $\mathbb{G}$,

1. recursion is guarded
2. continuations non-empty and non-$\bot$
3. $\mathbb{G} \xrightarrow{\mathcal{G}} G$
4. G is balanced

Definition (Grafting)

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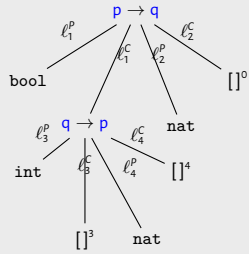

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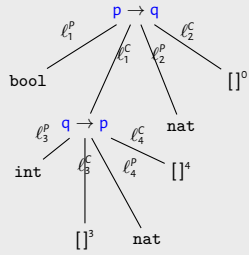

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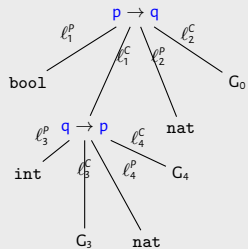

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Notation

Represent continuations of global and local type trees as *option lists* of *sort-tree* pairs: `p` $\to$ `q L` and $\oplus$`p! L`

Lemma (preservation of projection under reduction)

If we have

- $\texttt{G} \upharpoonright_{\texttt{p}} (\oplus \texttt{q! l}_1)$, $\texttt{G} \upharpoonright_{\texttt{q}} (\&\texttt{p? l}_2)$, $(\pi_{\texttt{T}}\ \texttt{l}_1)_{\texttt{n}} = \texttt{T}$, $(\pi_{\texttt{T}}\ \texttt{l}_2)_{\texttt{n}} = \texttt{T'}$, and $\texttt{G} \setminus \texttt{p} \xrightarrow{\texttt{n}} \texttt{q G'}$.

Then,

- $\texttt{G'} \upharpoonright_{\texttt{p}} \texttt{T}$ and $\texttt{G'} \upharpoonright_{\texttt{q}} \texttt{T'}$

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Lemma (consumption from projection and subtyping)

If

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- $\&\texttt{p?}\ \texttt{xs} \leqslant \&\texttt{p?}\ \texttt{l}_2$ with $(\texttt{xs})_{\texttt{n}} = (\texttt{s'},\texttt{T'})$, and
- $\oplus\texttt{q!}\ (+[\texttt{n}]\ (\texttt{s}, \texttt{T})) \leqslant \oplus\texttt{q!}\ \texttt{l}_1$.

Then,

- $\exists\ \texttt{G'}$ such that $\texttt{G} \setminus \texttt{p} \xrightarrow{\texttt{n}} \texttt{q}\ \texttt{G'}$.

Theorem (subject reduction)

Have

- $\vdash M : G$ and $M \longrightarrow M'$

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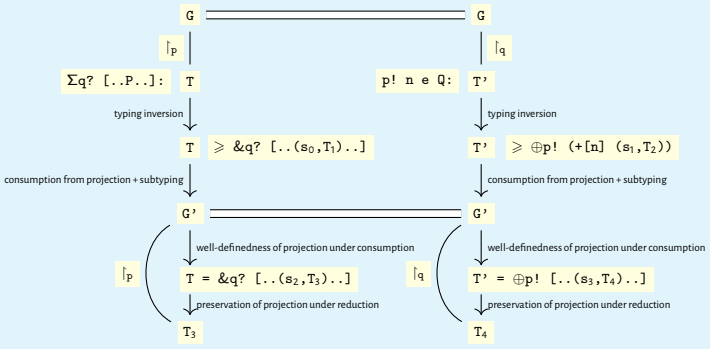
Proof sketch ([r-comm])

$$\begin{cases} (H) & \vdash \texttt{(p} \triangleleft \texttt{\&q? L | q} \triangleleft \texttt{} \oplus \texttt{p! n e Q | M): G}\,, \\ (Hn) & \texttt{L}_\texttt{n} \texttt{ = Some P}\,. \end{cases}$$

with `e` reduces into the value `v`, and the goal looks like

$$\begin{cases} (G_1) & \exists \texttt{ G'} \text{ such that } \vdash \texttt{(p} \triangleleft \texttt{ P[v/0] | q} \triangleleft \texttt{ Q | M): G'} \\ (G_2) & \texttt{G} \longrightarrow \texttt{G'}\,. \end{cases}$$

Proof sketch ([r-comm] rule)



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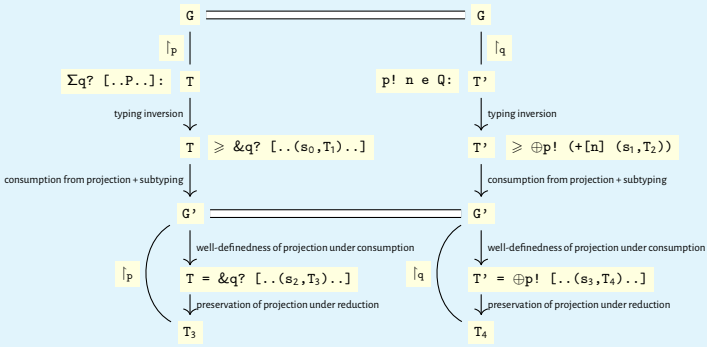

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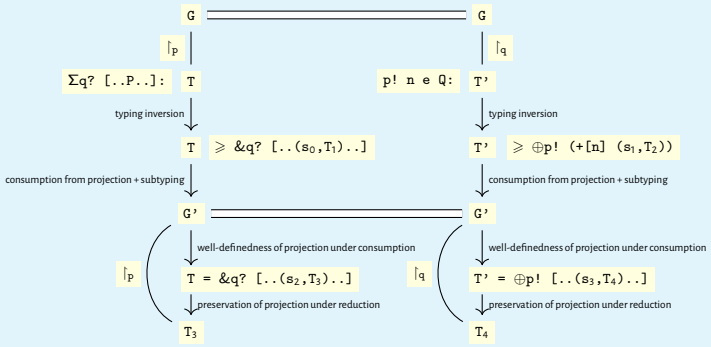

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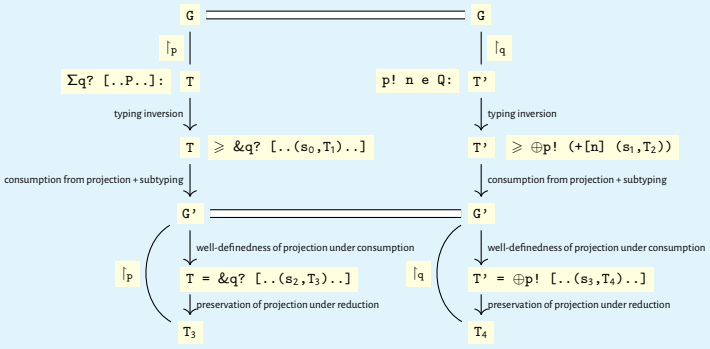

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Lemma (canonical forms for processes and sessions)

- Given $\vdash$ `M: (p → q xs)` . Then,
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Theorem (non-stuck)

If $\vdash$ `M: G` , then `M` does not get stuck.

Outline

Approach

- usual inductive declarations for session types

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Inductive global  : Type ≜
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- obviously, when `R` is empty

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Approach (use of `list` in `gtt` (cont'd))

- use of lists $\longrightarrow$ relaxing related definitions and (universal) proofs in Rocq
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- use `colists` or function types in `gtt`

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  allows for implementing "`translate`" as a cofixpoint

Theorem

if projecting a *well-formed* tree G onto a participant p results in trees T_1 and T_2, then $T_1 = T_2$

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- coinductive types modulo coinductive extensionality is equivalent to function types with functional extensionality

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- $\sim$16K lines of Rocq code

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- accessible at: `https://github.com/Apiros3/smpst-sr-smer`

Related & Future Work

Related:

1. Zooid [3]: certified multiparty communication in Rocq, ensuring deadlock-free, protocol-compliant execution
2. Multris [8]: Iris framework for local protocol consistency in multiparty concurrency, no global type guarantees
3. MPGV [10]: Linear λ-calculus + MPST; deadlock-free, with progress & preservation in separation logic of Iris
4. Ekici and Yoshida [4] formalise, in Rcoq, subtyping properties in asynchronous multiparty communication
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Future:

1. incorporating **full merging** into projections
2. full formal proof of **liveness** for synchronous MPST

Thanks! & Questions?

https://github.com/Apiros3/smpst-sr-smer

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