The Formal Conjectures repo

Moritz Firsching

Plan for the talk

What

Why

How

Who

What

Formal Conjectures is a collection of mathematical statements formalized in Lean/mathlib.

448 open conjectures and

334 solved problems, i.e. statements where an informal proof is available.

solved variants for open conjectures or from problem collections.

Mainly number theory and combinatorics currently.

Most statements require little definitions on top of mathlib to formulate them.

What: problem sources

- Erdős problems: collected by Thomas Bloom at https://www.erdosproblems.com
- Wikipedia
- ArXiv / journal papers
- Books
- Mathoverflow

Why: different aspects of the repo

- Benchmark
  - measure automatic proof generation and autoformalizaion
  - won’t saturate quickly
- comprehensive library containing *all* important open question
  - thereby guiding development of mathlib
  - clarify meaning of conjectures
- A high latency way to interact with automatic proving agents
  - can be viewed as a unified API

Community driven: let’s decide what conjectures are interesting!

How

Manually formalized, (autoformalization?)

Mitigations to avoid misformalization:

- manual review
- using automated theorem proving to find misformalization:
  If a statement or its opposite is trivially provable, then it is likely misformalized.
- Add simpler versions/test statements.

Example: remark for Erdős problem #730

OPEN

Are there infinitely many pairs of integers $n \neq m$ such that $\binom{2n}{n}$ and $\binom{2m}{m}$ have the same set of prime divisors?

#730: [EGRS75] number theory, binomial coefficients, base representations

A problem of Erdős, Graham, Ruzsa, and Straus [EGRS75], who believed there is 'no doubt' that the answer is yes.

For example $(87, 88)$ and $(607, 608)$. Those n such that there exists some suitable $m > n$ are listed as A129515 in the OEIS.

In every known example $m = n + 1$.

Example: remark for Erdős problem #730

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```
private abbrev S :=
  {(n, m) : ℕ × ℕ | n < m ∧ ((2*n).choose n).primeFactors = ((2*m).choose m).primeFactors}

/--
In every known example $(n, m) ∈ S$, we have $m = n + 1$.
-/
@[category research open, AMS 11]
theorem erdos_730.variants.delta_one (n m : ℕ) (h : (n, m) ∈ S) : m = n + 1 := by
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AlphaProof found a disproof: the counter example n = 10003 and m = 10005. 🎉

(See here for a larger numerical search: https://github.com/mo271/kummer)

Who: community contributions ❤️

- 43 contributors
- 26 returning contributors (i.e. > 1 commit)

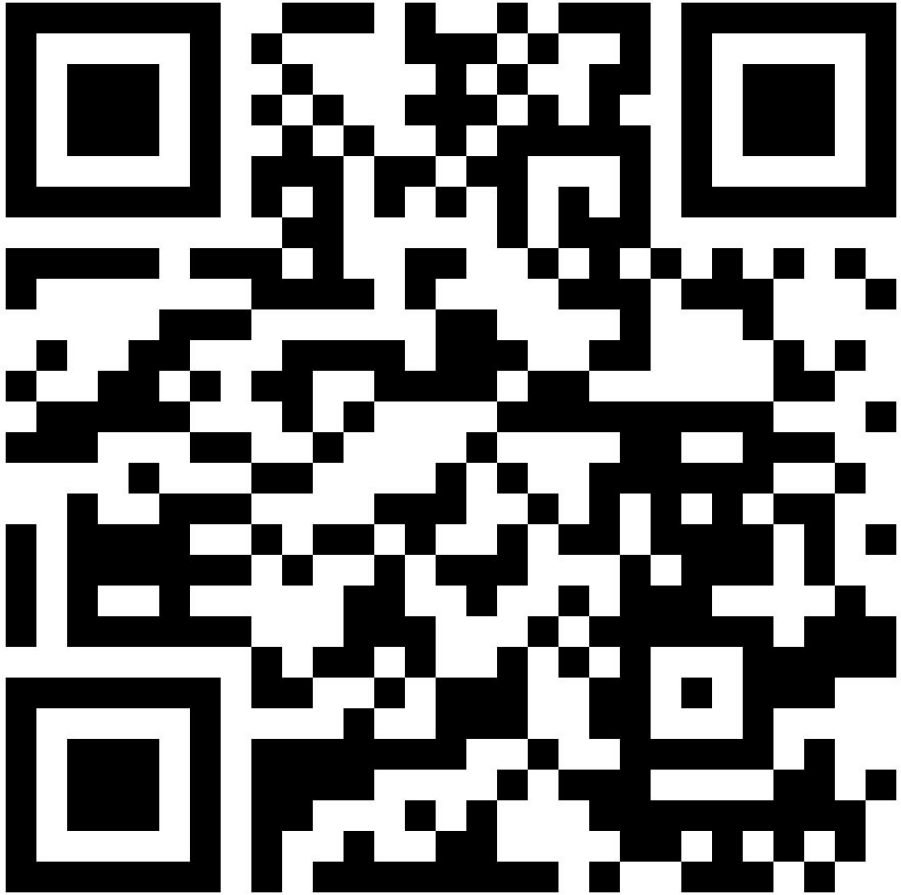
For some contributors their first lean experience!

Easy start for mathematicians not fluent in Lean

What are your favorite conjectures?!

Moritz Firsching
Paul Lezeau
Salvatore Mercuri
Calle Sönne
Eric Wieser (curr. at ITP 2025!!)
Bhavik Mehta
Reklle
Seewoo Lee
Junyan Xu
Divyanshu Ranjan
Bolton Bailey
peabrainiac
Anirudh Rao
Alex Kontorovich
Anthony Wang
...

Question/Discussion



github.com/google-deepmind/formal-conjectures