Extending SortPoly with Elimination Constraints in Rocq

The Rocqshop 2025, Reykjavik, Iceland

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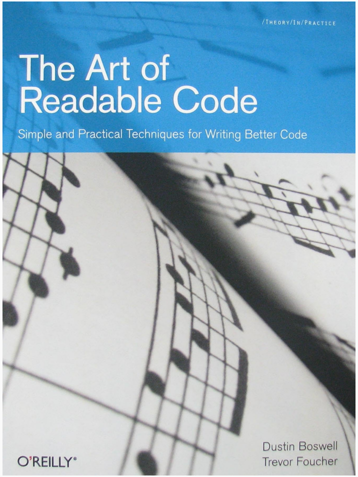
[4]LMF & Inria Saclay, Saclay, France

When You Stare Into The Code, it Stares Back At You



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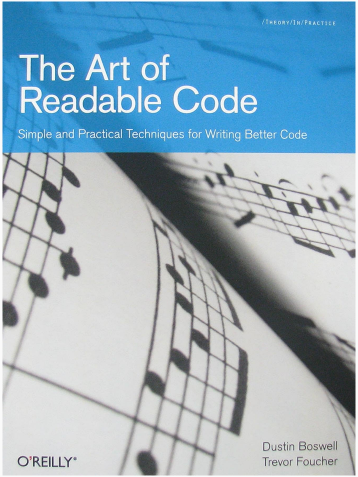

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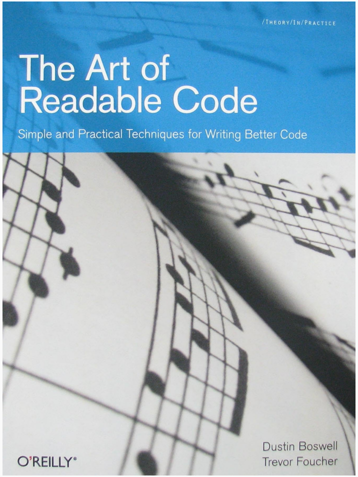

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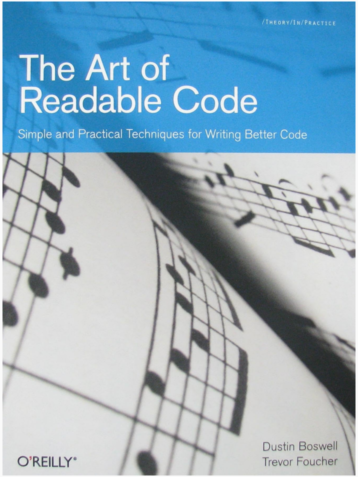

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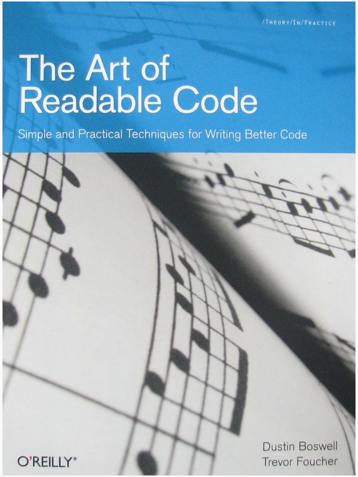

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Guess what? today's talk: duplications.

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Poiret et al. saved us from this world of suffering with SortPoly:

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Inductive sum@{sl sr s ; ul ur}
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But this is not enough to avoid duplication!

Arguably Worse Situation

With unbounded sort polymorphism: cannot define *e.g.*, a generic eliminator:

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Still need to declare the different eliminators "by hand".

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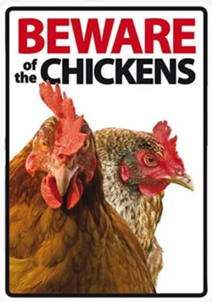

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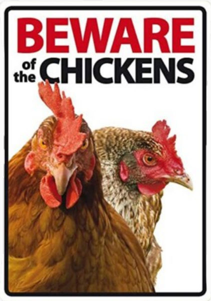

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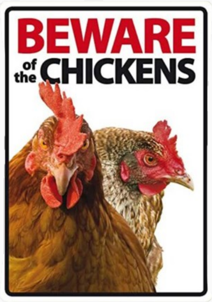

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This is the job of elimination constraints in $\mathrm{SortPol}\vec{\mathrm{y}}$.

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For `s`, `s'` two *sorts*:

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Study of the metatheory gives:

- consistency (under small condition), and
- transitivity.

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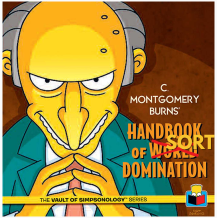

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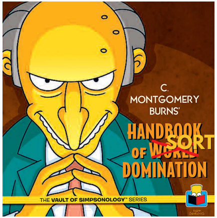

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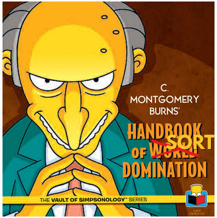

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Good news: in Rocq, `Type` is initial.

$\Longrightarrow$ only need to ensure domination.

It’s Even Getting User-Friendly

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Does not catch everything, *e.g.*, inconsistent sort eliminating to `Type`.

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Solution: one elaboration procedure to infer them all.

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Concretely, for the user:

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Phew, we won't have to go insane over that

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There is Even Actual Work Done

Implementation of elimination constraints in Rocq:

- reuse of the universe level graph for transitive closure,
- ad-hoc checks for dominant sorts (amortized constant complexity),
- ad-hoc checks to avoid introducing unwanted constraints,
- manual prohibition of `SProp` $\rightsquigarrow$ `s`.

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Expect some performance regressions in the monomorphic case:

- eliminability check through a graph,
- bigger structures at elaboration.

Rough Planning (There is a Strange Theme in These Titles, no?)

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- Develop the constraint graph, and plug it at the right places.
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Near future:

- Make use of dominant sorts in conversion tests.

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Any question(s)?