Informatics

University of Southampton

Finding Connections via Satisfiability Solving

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Joint work with

Michael Rawson (U. Southampton) & *Laura Kovács* (TU Wien)

The Two Worlds

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(Mostly) Efficient Reasoning	Construction (mostly) by enumeration/intuition
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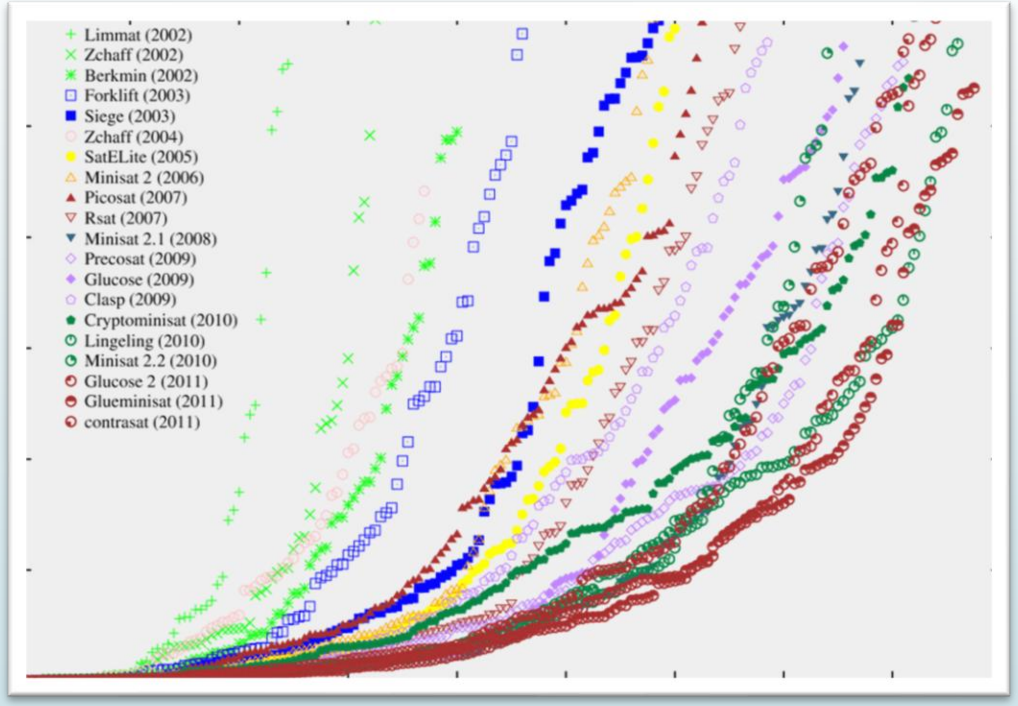
Theory of Proof Objects

One Step back: Why Combine them?

SAT Solvers are efficient



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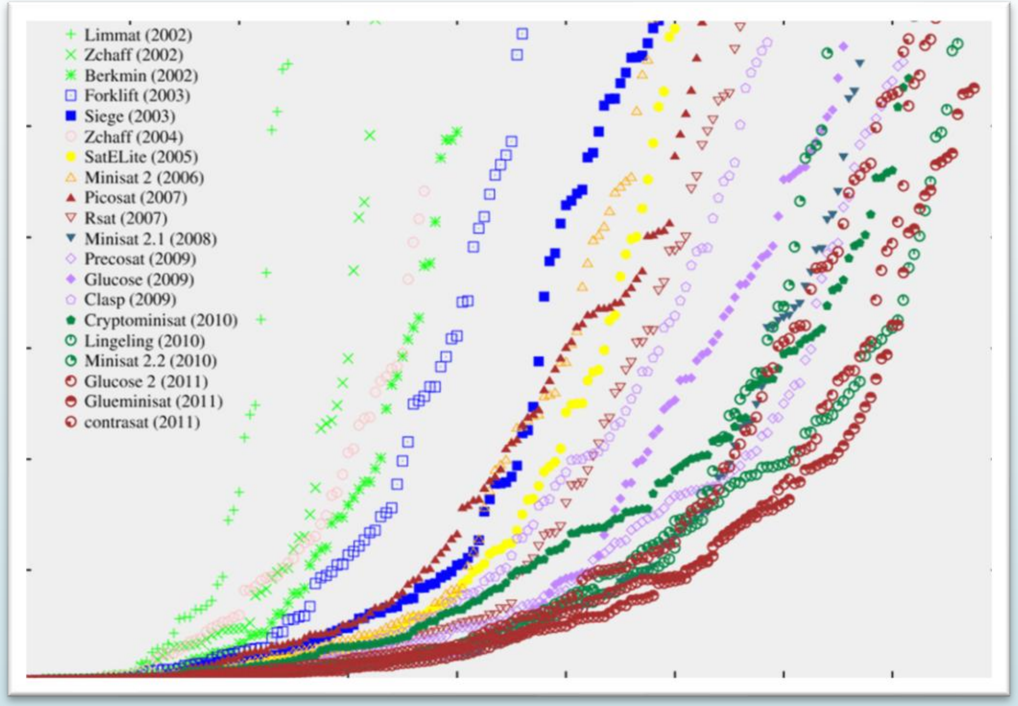

… and get more efficient every year

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- We **continued** on that end and here is what came out…

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- Iterative deepening & proof enumeration

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C_2: $\forall z \left(P(z) \vee P\big(f(z)\big)\right)$

$P(z_1)$	$\neg P(x_1)$	$\neg P(x_2)$
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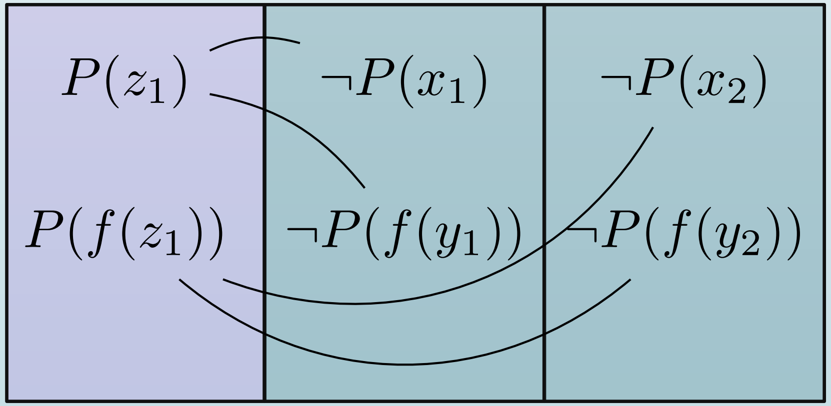

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C_1: $\forall x \forall y \left(\neg P(x) \vee \neg P(f(x))\right)$

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$\sigma(z_1) \mapsto f(y_1)$, $\sigma(x_1) \mapsto f(y_1)$,
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SAT/SMT Encodings for CC

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1. CC tableau trees (omitted for *this* talk)
2. **CC matrices with static sizes**
3. **CC matrices with dynamic sizes**

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➔ We check for spanningness and rule out spurious proofs on-demand

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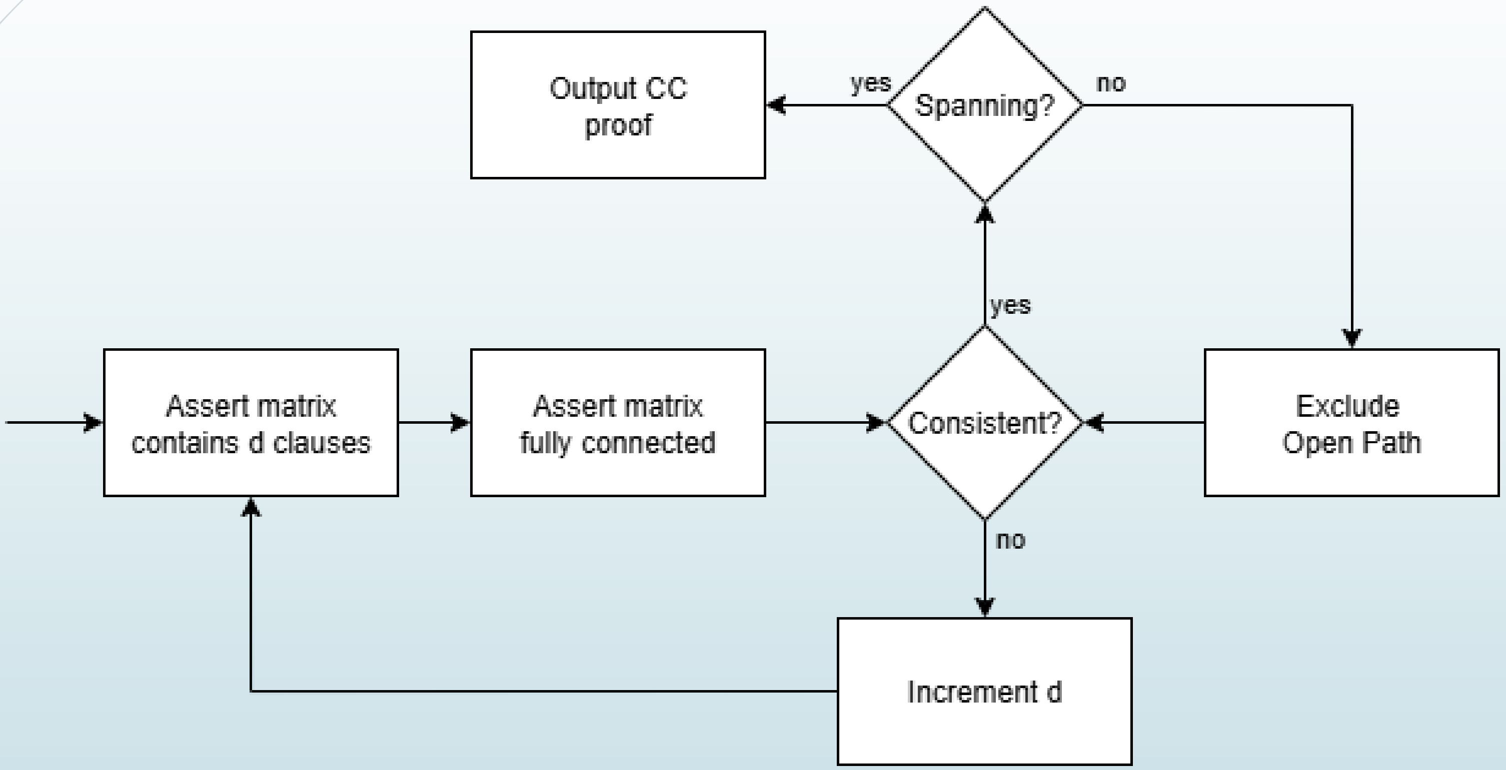

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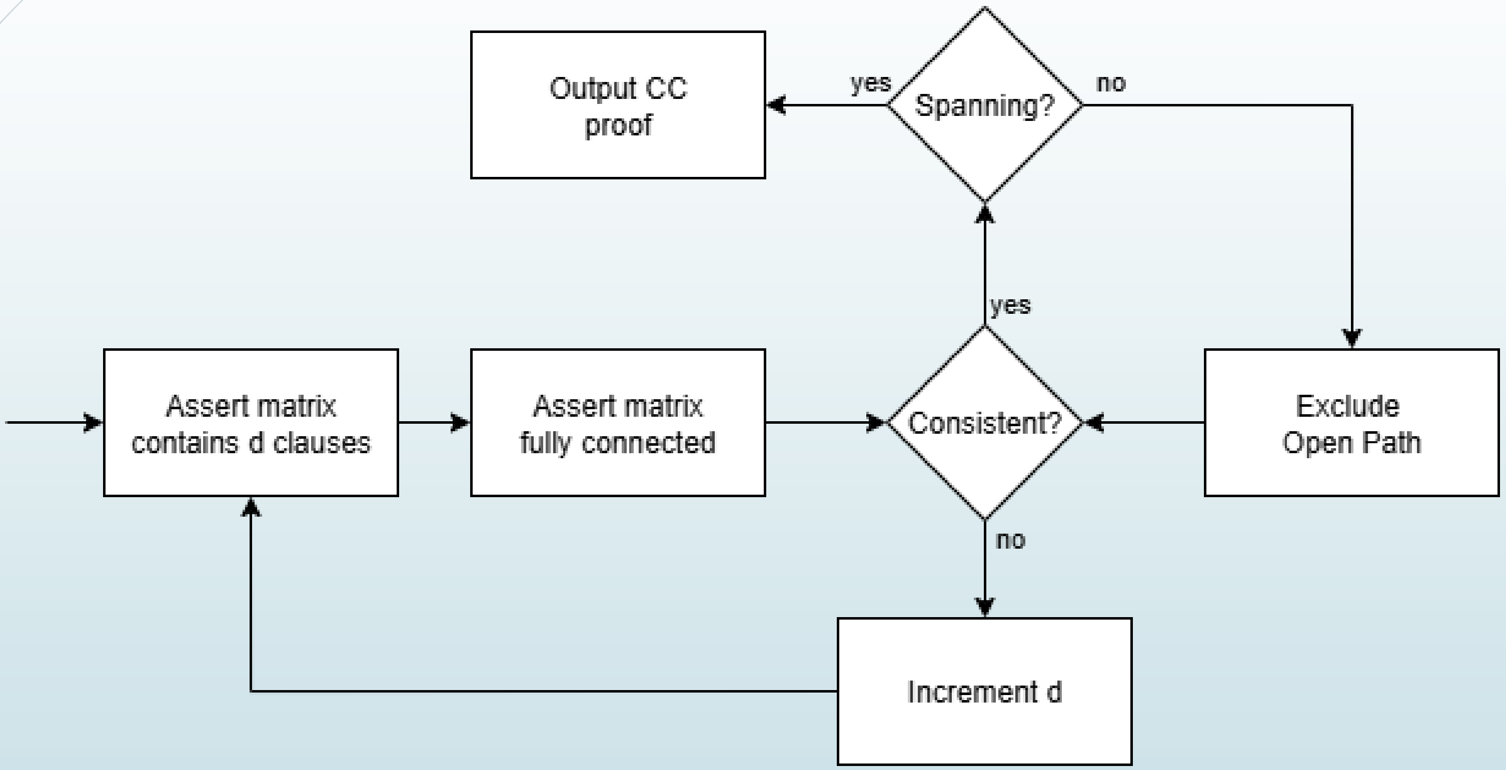

- **Encoding is lazy** (SMT style)
- Unification can be expressed using **algebraic datatypes** (ADT)

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     - i.e., enforces $f(z_1)$ unifies with x_1

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3. For every open path U and set of *d* selectors $\bar{S}$ we add

$$\bigwedge_{S \in \bar{S}} S \Rightarrow \bigvee_{\{L,K\} \subseteq U} \langle L \sim K \rangle$$

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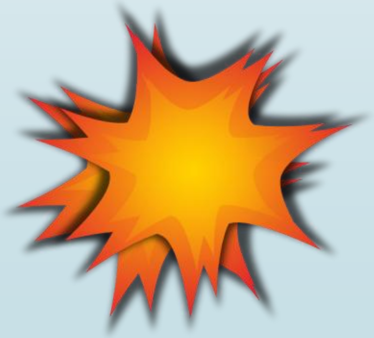
- However, incrementing *d* by one ➔ one more copy of *each clause*
  - Makes it even more explosive



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- „Connect to some literal instance of clause D or require more instances of D“

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$$P(f(x_1))$$

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$$S^1_{\neg P(x) \vee P(f(x))} \Rightarrow \Big(S^2_{\neg P(x) \vee P(f(x))} \wedge \big\langle P\big(f(x_1)\big) \sim P(x_2)\big\rangle\Big)$$

Incrementing Capacity

- Consider the clause set

$$\{\, P(a), \quad \neg P(x) \vee P\big(f(x)\big), \quad \neg P(a) \,\}$$

$$P(a) \text{ —— } \neg P(x_1)$$

$$P(f(x_1))$$

$$\mu\big(P(a)\big) \mapsto 1$$
$$\mu\Big(P(x) \vee \neg P\big(f(x)\big)\Big) \mapsto 1$$
$$\mu\big(\neg P(a)\big) \mapsto 0$$
$$\{\, S^2_{\neg P(x) \vee P(f(x))}, \quad S^1_{\neg P(a)} \,\}$$

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Incrementing Capacity

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$$P(a) \;—\; \neg P(x_1) \qquad \neg P(x_2)$$

$$P(f(x_1)) \qquad P(f(x_2))$$

$$\mu\big(P(a)\big) \mapsto 1$$

$$\mu\Big(P(x) \lor \neg P\big(f(x)\big)\Big) \mapsto 2$$

$$\mu\big(\neg P(a)\big) \mapsto 0$$

Incrementing Capacity

➔ Fair incrementation of clauses in unsat cores

- Consider the clause set

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$P(a)$ —— $\neg P(x_1)$ $\qquad$ $\neg P(x_2)$

$P(f(x_1))$ $\qquad$ $P(f(x_2))$

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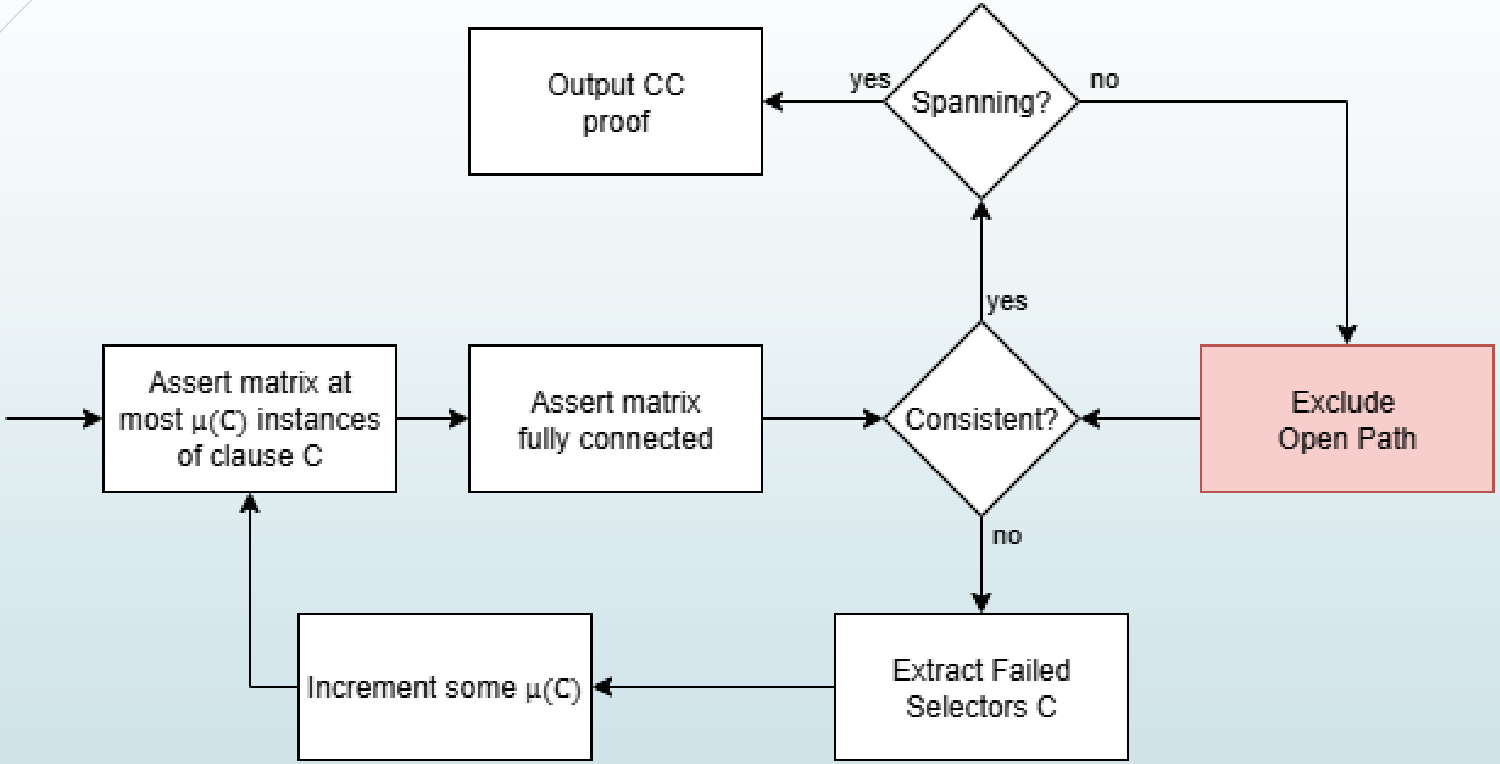
Dynamic Matrix Sizing Idea



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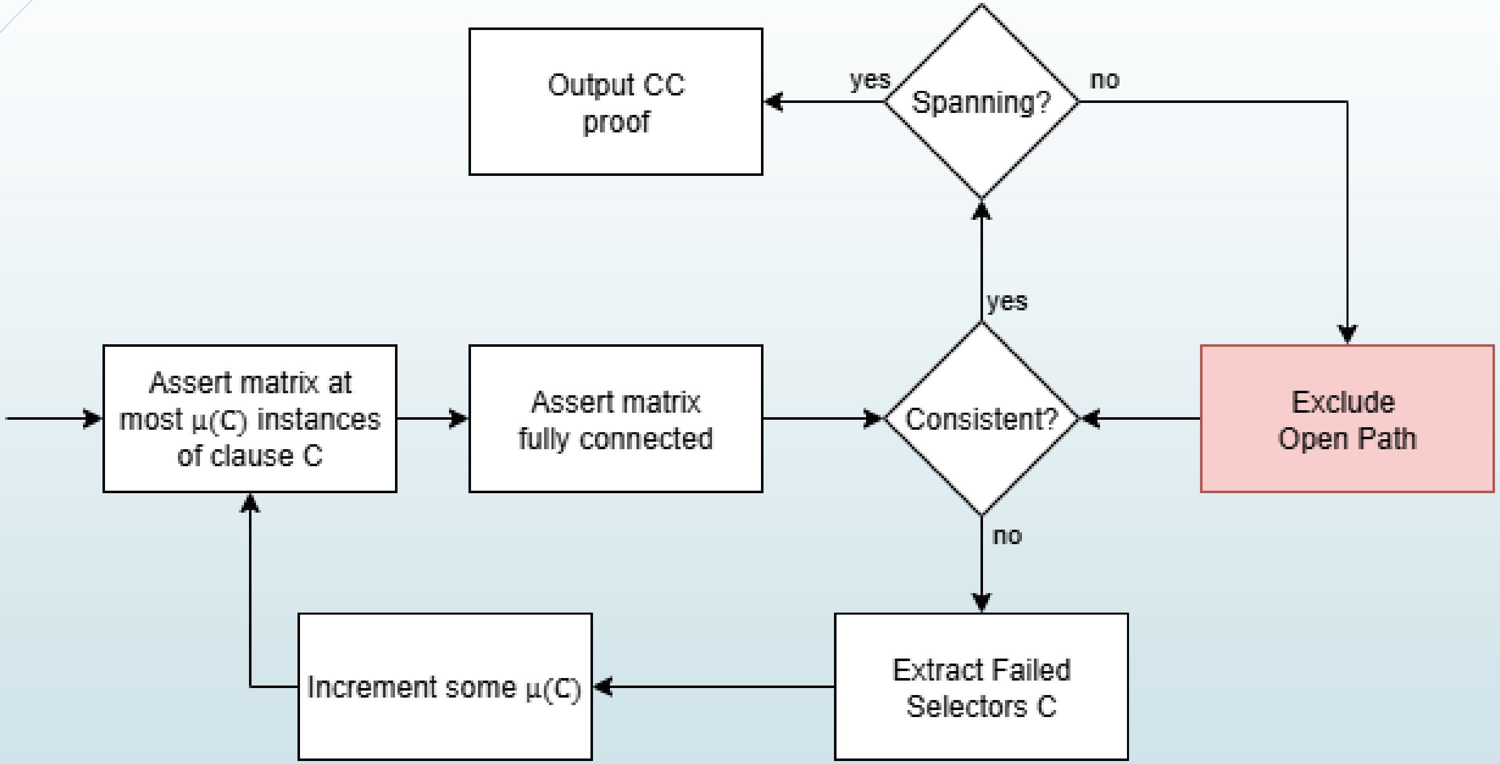

- Eliminating open paths is **broken** now

Why Broken?

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- The **at most** – instead of exactly – breaks it!

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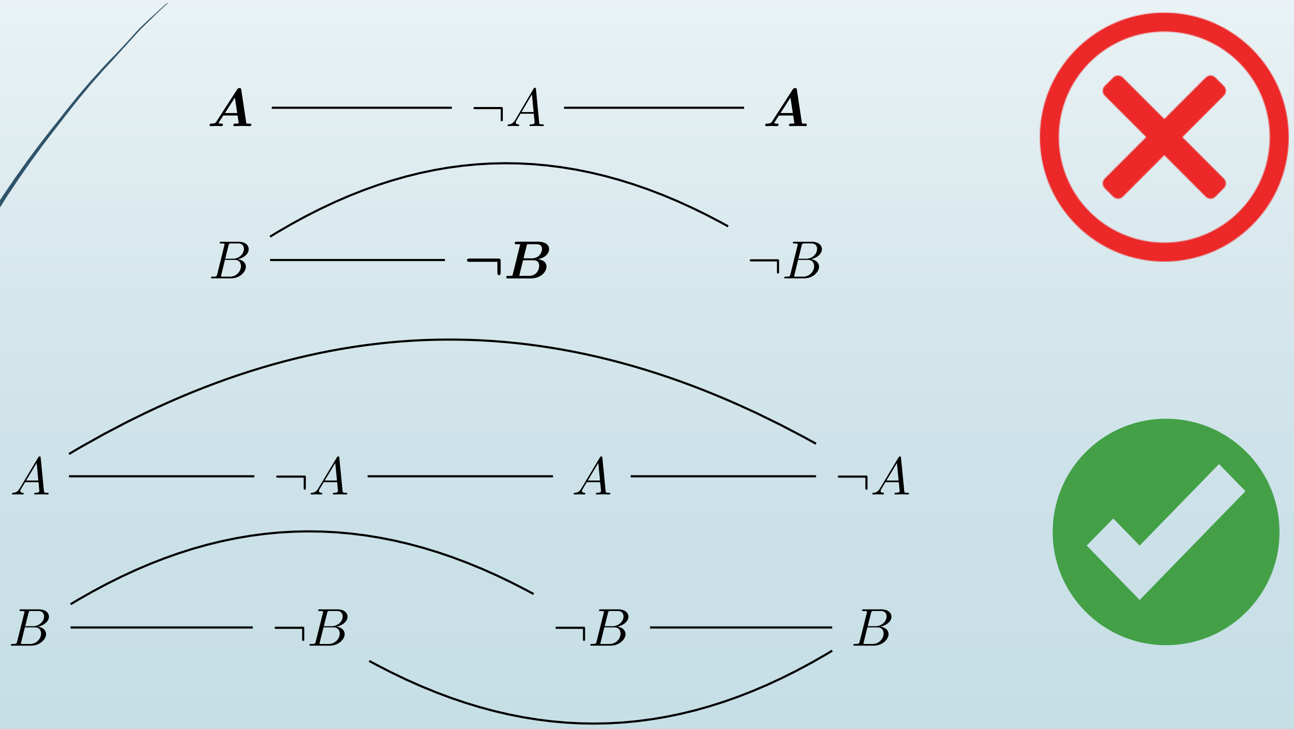

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$$\bigwedge_{S\in\bar{S}} S \Rightarrow \bigvee_{\{L,K\}\subseteq U} \langle L \sim K \rangle$$

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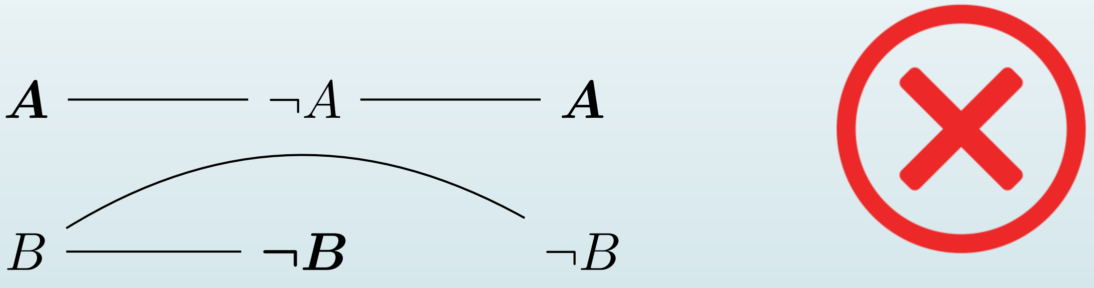
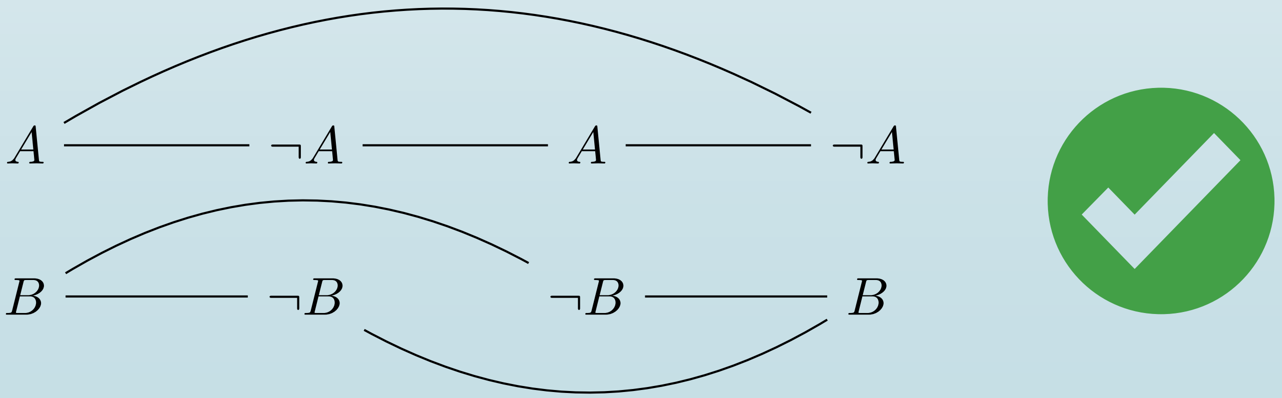
{ A ∨ B, ¬A ∨ ¬B, A ∨ ¬B, ¬A ∨ B }





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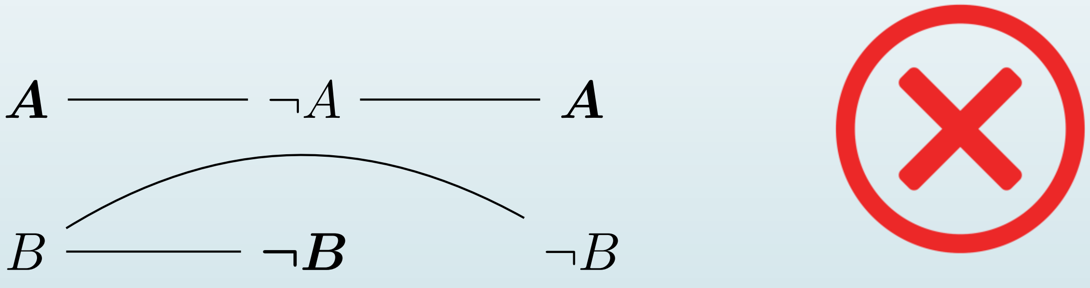
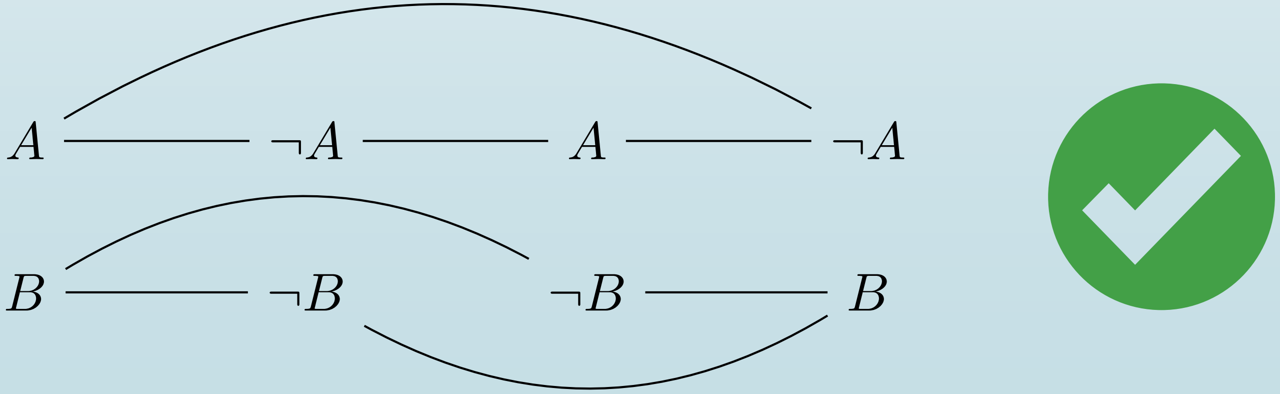


$$(S^1_{A\vee B} \wedge S^1_{\neg A\vee\neg B} \wedge S^1_{A\vee\neg B}) \Rightarrow \bot$$



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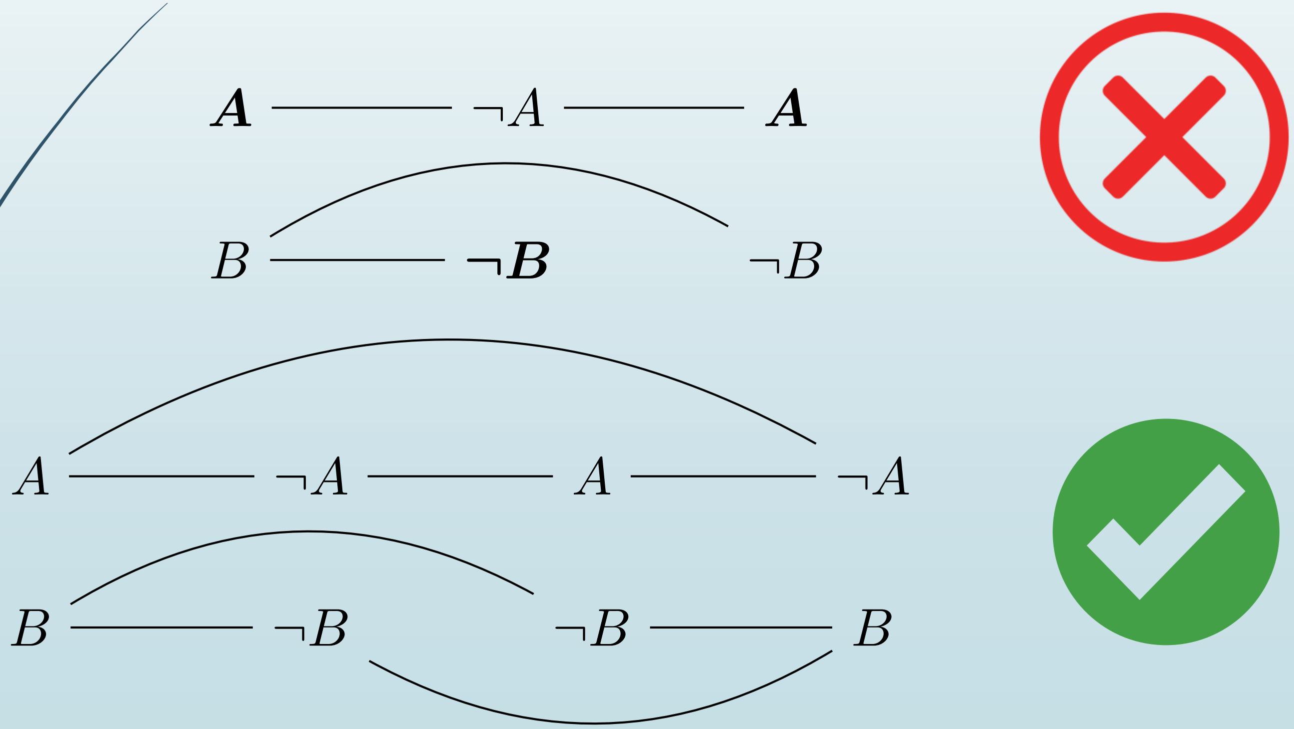

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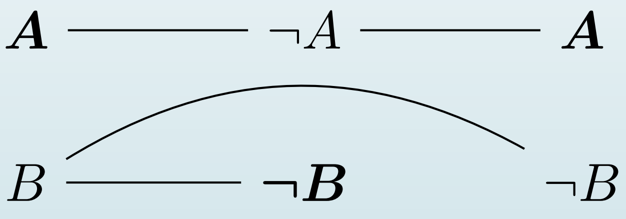

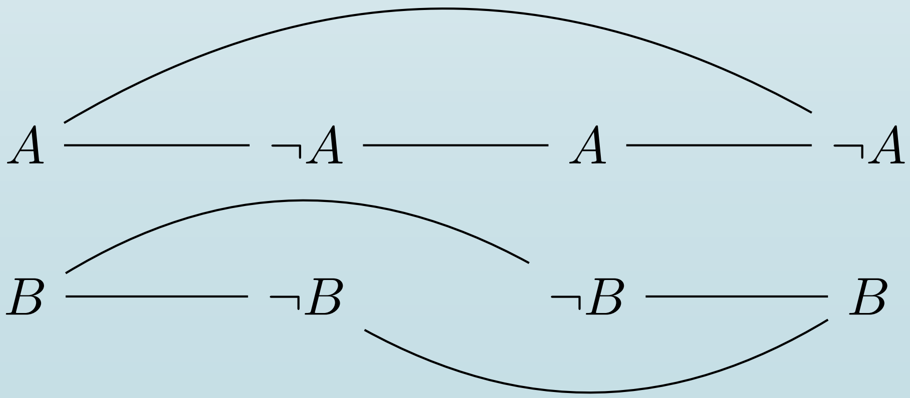

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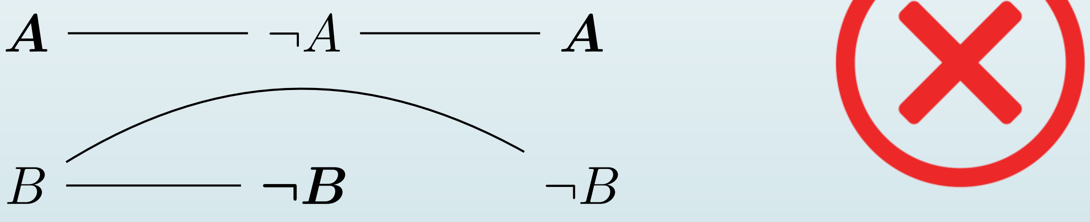

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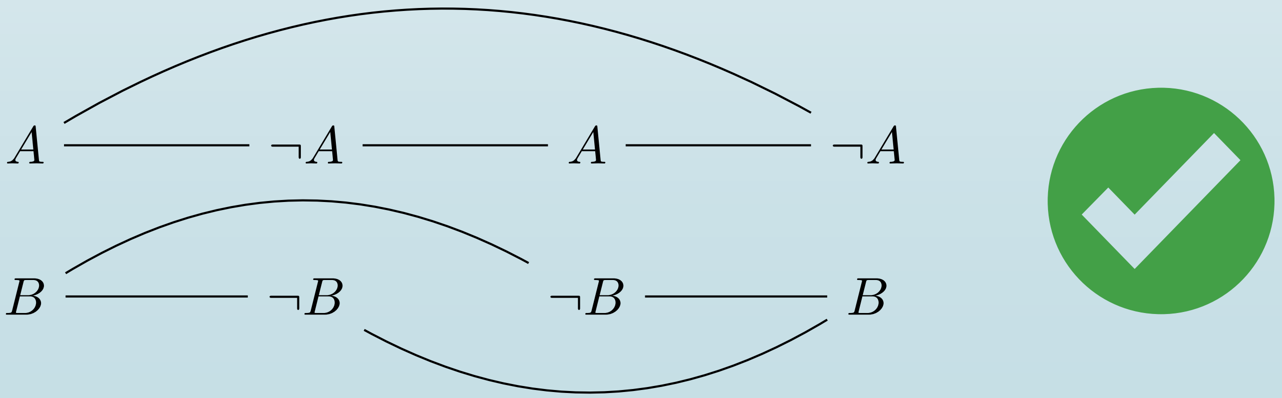

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$$S_C^{i+1} \Rightarrow S_C^{i}$$

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- Prototype *UpCoP* using both *CaDiCal* (SAT) and *Z3* (SMT) backend
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Enc.	$\mathcal{E}_{\text{M}}$		$\mathcal{E}_{\text{U}}$		$\mathcal{E}_{\text{H}}$		
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Huch! What went wrong??

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$$\boxed{P(a)} \qquad \boxed{\begin{array}{ccc} \neg Q(x_1) & \neg Q(x_2) & \neg Q(x_3) \\ Q(f(x_1)) & Q(f(x_2)) & Q(f(x_3)) \end{array}}$$

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No SAT-guided CC ☹

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Stay tuned for the next lecture ☺

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Questions?