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2022 LOGIC COLLOQUIUM: SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Reykjavik University
Reykjavik, Iceland

June 27 – July 1, 2022

Abstract of the invited 33rd Annual Gödel Lecture

▶ PATRICIA BLANCHETTE, *Formalism in Logic*.
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Logic became 'formal' at the end of the 19th century primarily in pursuit of deductive rigor within mathematics. But by the early 20th century, a formal treatment of logic had become essential to two new streams in the current of logic: the collection of crucial 'semantic' notions surrounding the idea of categoricity, and the project of examining the tools of logic themselves, in the way that's crucial for the treatment of completeness (in its various guises). This lecture discusses the variety of different tasks that have been assigned the notion of formalization in the recent history of logic, with an emphasis on some of the ways in which the distinct purposes of formalization are not always in harmony with one another.

Abstract of invited tutorials

▶ LIBOR BARTO,*Algebra and Logic in the Complexity of Constraints*.
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What kind of mathematical structure in computational problems allows for efficient algorithms? This fundamental question now has a satisfactory answer for a rather broad class of computational problems, so called fixed-template finite-domain Constraint Satisfaction Problems (CSPs). This answer, due to Bulatov and Zhuk, stems from the interplay between algebra and logic, similar to the classical connection between permutation groups and first-order definability.

Th aim of this tutorial is to explain this algebra-logic interplay, show how it is applied in CSPs, and discuss some of the major research directions.

▶ LUCA SAN MAURO, *Computable reductions of equivalence relations*.
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The study of the complexity of equivalence relations has been a major thread of research in diverse areas of logic. A reduction of an equivalence relation E on a domain X to an equivalence relation F on a domain Y is a function $f : X \to Y$ which induces an injection on the quotient sets, $X_{/E} \to Y_{/F}$. In the literature, there are two main definitions for this reducibility.

- In descriptive set theory, *Borel reducibility* is defined by assuming that X and Y

are Polish spaces and f is Borel.
- In computability theory, *computable reducibility* is defined by assuming that X and Y coincide with the set of natural numbers and f is computable.

The theory of Borel equivalence relations is a central field of modern descriptive set theory and it shows deep connections with topology, group theory, combinatorics, model theory, and ergodic theory – to name a few. On the other hand, computable reducibility dates back to the 1970s and it found remarkable applications in a diverse collection of fields, including the theory of numberings, proof theory, computable structure theory, combinatorial algebra, and theoretical computer science.

Despite the clear analogy between the two notions, for a long time the study of Borel and computable reducibility were conducted independently. Yet, it is rapidly emerging a theory of computable reductions which blends ideas from both computability theory and descriptive set theory. This tutorial will overview such a theory. We will present computable, or computably enumerable, analogs of fundamental concepts from the Borel theory (e.g., benchmark equivalence relations, dichotomy results, orbit equivalence relations, the Friedman-Stanley jump), highlighting differences and similarities between the Borel and the computable setting. We will also report on recent progress in the abstract study of computable reducibility, focusing on both local structures of equivalence relations of given complexity and the global structure of all equivalence relations on the natural numbers.

Abstracts of invited plenary lectures

► TUNA ALTINEL, *On the actions of finite permutation groups on groups of finite Morley rank.*

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Ever since the work of Borovik and Cherlin on permutation groups of finite Morley rank ([4]), there has been growing interest in faithful actions finite groups on various types of groups of finite Morley rank. This interest is due to various motivations: classifying highly generically transitive actions on sets ([1]), highly generically transitive representations ([]), definable actions of finite groups on groups of finite Morley rank, automorphisms of groups of finite Morley rank. In my talk, I will give an overview of these lines of research and detail a recent result joint with Joshua Wiscons on lower bounds in the case of faithful actions of the alternating group on a nonsolvable group of finite Morley rank that does not contain involutions.

[1] Tuna Altinel and Joshua Wiscons, *Recognizing* PGL_3 *via generic* 4-*transitivity*, ***Journal of the European Mathematical Society***, vol. 20 (2018), no. 6, pp. 1525–1559.

[2] Ayşe Berkman and Alexandre Borovik *Groups of finite Morley rank with a generically sharply multiply transitive action*, ***Journal of Algebra***, vol. 368 (2012), no. X, pp. 237–250.

[3] Ayşe Berkman and Alexandre Borovik *Groups of finite Morley rank with a generically multiply transitive action on an abelian group*, arXiv:2107.09997 (preprint).

[4] Alexandre Borovik and Gregory Cherlin, *Model theory with applications to algebra and analysis. Vol. 2*, ***Model theory with applications to algebra and analysis. Vol. 2*** (H. Dugald Macpherson, editors), Cambridge Univ. Press, Cambridge, 2008, pp. 30–50.

[5] Luis Jaime Corredor and Adrien Deloro and Joshua Wiscons, Sym(n)-

and Alt(n)*-modules with an additive dimension*, ***Arxiv***, (2022), eprint 2111.11498.

► ANTON FREUND, *The uniform Kruskal theorem: a bridge between finite combinatorics and abstract set existence.*
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An important theorem of J. Kruskal states that any infinite sequence $t_0, t_1, \dots$ of finite trees admits $i < j$ such that t_i embeds into t_j. As shown by H. Friedman, this theorem – and even a 'finitized' corollary – is unprovable in predicative axiom systems, such as the theory ATR_0 from reverse mathematics. This is one of the most convincing mathematical examples for the incompleteness phenomenon from Gödel's theorems.

The 'minimal bad sequence lemma' due to C. Nash-Williams provides a particularly elegant proof of Kruskal's theorem. By a result of A. Marcone, this lemma is equivalent to the impredicative principle of Π^1_1-comprehension, over a weak base theory from reverse mathematics. Kruskal's theorem itself cannot be equivalent to this principle, as its quantifier complexity is too low. This suggests the following question:

In which sense can we view Kruskal's theorem as the concrete 'shadow' of an abstract set existence principle?

To suggest an answer, I will present joint work with M. Rathjen and A. Weiermann [4], which shows that Π^1_1-comprehension is equivalent to a uniform version of Kruskal's theorem (with general recursive data types at the place of trees). Together with the aforementioned result of Marcone, this confirms the intuition that minimal bad sequences provide 'the' canonical proof.

An analogous equivalence [2] has been established between Π^1_1-transfinite recursion, a minimal bad sequence result of I. Kříž, and a uniform version of Friedman's extended Kruskal theorem with ordinal labels and gap condition. The results rely on previous work [1, 3] that connects the 'concrete' viewpoint of ordinal analysis with the more 'abstract' setting of reverse mathematics.

The results and proofs will be presented on an intuitive level. Beyond the specific case of Kruskal's theorem, the hope is to shed some light on a remarkable phenomenon in modern mathematics: that concrete statements about finite objects are sometimes proved via abstract and infinite ones.

[1] Anton Freund, Π^1_1*-comprehension as a well-ordering principle*, ***Advances in Mathematics***, vol. 355 (2019), article no. 106767, 65 pp.

[2] Anton Freund, *Reverse mathematics of a uniform Kruskal-Friedman theorem*, arXiv:2112.08727 (preprint), 25 pp.

[3] Anton Freund and Michael Rathjen, *Well ordering principles for iterated* Π^1_1*-comprehension*, arXiv:2112.08005 (preprint), 67 pp.

[4] Anton Freund, Michael Rathjen and Andreas Weiermann, *Minimal bad sequences are necessary for a uniform Kruskal theorem*, ***Advances in Mathematics***, vol. 400 (2022), article no. 108265 , 44 pp.

► GUNTER FUCHS, *Blurry HOD – a sketch of a landscape.*
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Classically, a set is ordinal definable if it is the unique object satisfying a formula with ordinal parameters. Generalizing this concept, given a cardinal κ, I call a set $<\kappa$-blurrily definable if it is one of less than κ many objects satisfying a formula with ordinal parameters (called a $<\kappa$-blurry definition). By considering the hereditary versions of this notion, one arrives at a hierarchy of inner models, one for each cardinal κ: the collection of all hereditarily $<\kappa$-blurrily ordinal definable sets, which I call $<\kappa$-HOD. In a ZFC-model, this hierarchy spans the entire spectrum from HOD to V.

The special cases $\kappa = \omega$ and $\kappa = \omega_1$ have been previously considered, but no systematic study of the general setting has been done, it seems. One main aspect of the study is the notion of a leap, that is, a cardinal at which a new object becomes hereditarily blurrily definable. The talk splits into two parts: first, the ZFC-provable properties of blurry HOD, which are surprisingly rich, and second, the effects of forcing on the structure of blurry HOD and the achievable leap constellations.

▸ PAWEŁ M. IDZIAK,*Complexity of equations solving – kith and kin.*
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The talk is intended to present latest achievements in searching what structural algebraic conditions a finite algebra $\mathbf{A}$ has to satisfy in order to have a polynomial time algorithm that decides if an equation $\mathbf{s}(x_1, \ldots, x_n) = \mathbf{t}(x_1, \ldots, x_n)$, where $\mathbf{s}, \mathbf{t}$ are polynomials over $\mathbf{A}$, has a solution in $\mathbf{A}$.

Several connections to modular circuits CC^0 of constant depth will be discussed. Most of the results are obtained together with Piotr Kawałek, Jacek Krzaczkowski or Armin Weiß.

▸ JULIETTE KENNEDY, *Do syntactic features supervene on semantic ones in foundations of mathematics? A few starting points.*
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The practice of foundations of mathematics is built around a firm distinction between syntax and semantics. But how stable is this distinction, and is it always the case that semantically presented mathematical objects, in the form e.g. of a model class, might give rise to a "natural logic" in which the model class is definable? Can a logic without a syntax be considered a logic at all? In this talk I will investigate different scenarios from set theory and model theory in which an investigation of the notion of an implicit or internal logic or syntax becomes possible. I will close by discussing some historical issues raised by Blanchette [1], Goldfarb [2] and others having to do with the relation between having a precise syntax and the development of metamathematics, in early foundational practice.

[1] Patricia Blanchette, *From Logicism to Metatheory*, ***The Palgrave Centenary Companion to Principia Mathematica*** Bernard Linsky and Nicholas Griffin , editors), Palgrave Macmillan, Basingstoke, United Kingdom, 2013, pp.59–78.

[2] Warren Goldfarb, *Logic in the Twenties: the Nature of the Quantifier*, ***The Journal of Symbolic Logic***, vol. 44 (1978), no. 3, pp. 351–368.

▸ KAREN LANGE,*Classification via effective lists.*
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"Classifying" natural collection of structures is a common goal in mathematics. Providing a classification can mean different things, e.g., identifying a set of invariants that settle the isomorphism problem or creating a list of all structures of a given kind without repetition of isomorphism type. Here we discuss recent work on classifications of the latter kind from the perspective of computable structure theory. We'll consider natural classes of computable structures such as vector spaces, equivalence relations, algebraic fields, and trees to better understand the nuances of classification via effective lists and its relationship to other forms of classification in this setting.

► ALEXANDER G. MELNIKOV, *Primitive recursive mathematics*.
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In my talk I will discuss the current state of the rapidly developing filed of 'primitive recursive' mathematics. The subject has many different aspects. The main motivation of this framework is to understand the role of unbounded search in computable mathematics: either eliminate it when possible, or prove that without the unbounded search the result fails. Also, primitive recursion serves as a 'bridge' between the more abstract Turing computable mathematics and the perhaps more applicable polynomial-time and automatic algebra and analysis.

Over that past several years, investigations into this direction have uncovered many deep technical issues and results that were completely 'invisible' in the more general Turing computable algebra, analysis, and infinite combinatorics. Some recent results of this sort simply have no direct analogy in computable structure theory. In my talk I will emphasise those results and research directions in primitive recursive mathematics that either lead to counter-intuitive results or give new insights into other branches of effective mathematics. In particular, connections with automatic structure theory and reverse mathematics will be mentioned.

► MORITZ MÜLLER,*Automating Resolution is NP-hard*.
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Together with Albert Atserias we showed that it is NP-hard to find a Resolution refutation that is at most polynomially longer than a shortest one. The talk presents this result in its historical context.

► FEDOR PAKHOMOV,*Limits of applicability of Gödel's second incompleteness theorem*.
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The celebrated Gödel's second incompleteness theorem is the result that roughly speaking says that no strong enough consistent theory could prove its own consistency. In this talk I will first give an overview of the current state of research on the limits of applicability of the theorem. And second I will present two recent results: first is due to me [1] and the second is due to Albert Visser and me [2]. The first result is an example of a weak natural theory that proves the arithmetization of its own consistency. The second result is a general theorem with the flavor of Second Incompleteness Theorem that is applicable to arbitrary weak first-order theories rather than to extension of some base system. Namely the theorem states that no finitely axiomatizable first-order theory one-dimensionally interprets its own extension by predicative comprehension.

[1] Fedor Pakhomov, *A weak set theory that proves its own consistency*, arXiv:1907.00877 (preprint).

[2] Fedor Pakhomov and Albert Visser, *Finitely axiomatized theories lack self-comprehension*, arXiv:2109.02548 (preprint).

► FRANÇOISE POINT, *On differential expansions of topological fields*.
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A. Tarski, A. Robinson and A. Macintyre have described languages for which real-closed fields, algebraically closed valued fields, p-adically closed fieds admit quantifier elimination (and as a consequence one has a good understanding of definable sets in these structures). In particular, these structures are respectively o-minimal, C-minimal, p-minimal (more generally of dp-rank 1). More recently one has described satisfactory languages for which the corresponding theories admit elimination of imaginaries. In his work on Shelah's conjecture on fields with the non independence property (NIP), W. Johnson has shown that a field of dp-rank 1, which is not strongly minimal can be endowed with a definable field topology.

Differential expansions of (topological) fields of characteristic 0, where there is a priori no interactions between the derivation and the topology, have been first considered by M. Singer in the case of real-closed fields and he showed that the theory of differential ordered fields has a model companion. This was later generalized by M. Tressl in the class of large fields (a class of fields introduced by F. Pop).

In this talk, we consider the following setting. Given a large field of characteristic 0 endowed with a definable field topology and its theory T, we denote by T_δ the theory of differential expansions of models of T by a derivation δ (satisfying the usual axiom: $\delta(x+y) = \delta(x) + \delta(y) \wedge \delta(xy) = \delta(x)y + x\delta(y)$). Under some further conditions on definable subsets in models of T, we show the following. The class of existentially closed models of T_δ is first-order axiomatisable by a theory T_δ^*. Properties such as: quantifier elimination, the NIP property, elimination of imaginaries transfer from T to T_δ^*. In order to show the last result, we first prove a cell decomposition theorem for models of T, applying a similar strategy as for topological fields of dp-rank 1 due to P. Simon and E. Walsberg and then we show that there are no new open definable sets in models of T_δ^*. This approach can be applied to certain theories of pairs of models of T. These results were obtained in collaboration with N. Guzy and P. Cubides Kovacsics.

Then, using that the theories T we consider, are geometric theories (the topological dimension is well-behaved), we pursue our analysis to describe finite-dimensional definable groups in models of T_δ^*. We relate them to definable groups in models of T, using Weil's approach to recover an algebraic group from generic data. This last part is ongoing work with A. Pillay and K. Peterzil.

▶ ANDREA VACCARO, *Games on classifiable C*-algebras.*
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One of the major themes of research in the study of C*-algebras, over the last decades, has been Elliott's program to classify separable nuclear C*-algebras by their tracial and K-theoretic data, customarily represented in the so-called Elliott Invariant. In this talk I will analyze some subclasses of algebras (such as approximately finite C*-algebras) which fall within the scope of Elliott Classification Program from the perspective of infinitary continuous logic. More specifically, I will discuss how the techniques developed to classify nuclear C*-algebras can be combined with metric analogues of Ehrenfeucht–Fraïssé games, allowing to reduce the study of elementary equivalence between C*-algebras to the analogous relation on the discrete structures (groups and ordered groups) composing the Elliott Invariant. I will moreover show how this reduction can be employed to build classes of classifiable C*-algebras of arbitrarily high Scott rank.

Abstracts of invited talks in the Special Session on Computer Science Logic

▶ THORSTEN ALTENKIRCH, *Should Type Theory replace Set Theory as the Foundation of Mathematics?*
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Set theory in the form of Zermelo-Fraenkel's axiomatic set theory is usually considered the standard foundation of Mathematics. Type Theory which is based on the static notion of types is an alternative offers many advantages: the notion of a type seems to be closer to mathematical practice, types hides implementation details which enables Voevodky's univalence principle, and it is supported by a number of implementations providing the base for formal developments.

▶ SUSANNA F. DE REZENDE, *Proofs, circuits, and total search problems.*
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Many recent results in both propositional proof complexity and boolean circuit complexity have been enabled, either directly or indirectly, by a deeper understanding of proofs and circuits as a consequence of viewing them through the lens of *total search problems*, and by the development of *query-to-communication lifting theorems*, which show that in certain scenarios query complexity lower bounds can be "lifted" to communication lower bounds. Such results include explicit strongly exponential lower bounds on monotone formula complexity, separations between the $\mathsf{mon\text{-}AC}^i$ and the $\mathsf{mon\text{-}NC}^i$ hierarchies, new techniques for proving lower bounds on the size of monotone circuits and of cutting planes proofs, exponential lower bounds on the size of cutting planes proofs for random CNF formulas, the resolution of the Alon-Saks-Seymour problem, and many others.

This talk will focus on characterizations of proofs and circuits using the theory of total search problems (TFNP), expanding on classical results in complexity theory such as the characterization of circuit depth by Karchmer-Wigderson games, and the equivalence between tree-like Resolution and decision trees. We will also discuss how *lifting theorems* and *feasible interpolation* provide a connection between the query and communication complexity of certain search problems, and how this perspective suggests a whole program for further research.

▶ FABIO MOGAVERO, *Alternating (In)Dependence-Friendly Logic.*
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Informational independence is a phenomenon that emerges quite naturally in *game theory*, as players in a game make moves based on what they know about the state of the current play [8]. In games such as Chess or Go, both players have *perfect information* about the current state of the play and the moves they and their adversary have previously made. For other games, like Poker and Bridge, the players have to make decisions based only on *imperfect information* on the state of the play. Given the tight connection between games and logics, think for instance at *game-theoretic semantics* [5, 4, 1], a number of proposals have been put forward to reason with or about informational independence, most notably, *Independence-Friendly Logic* [2], *Dependence Logic* [7], and logics derived thereof.

Independence-Friendly Logic (IF) was originally introduced by Hintikka and Sandu [2], and later extensively studied, e.g., in [6], as an extension of *First-Order Logic* (FOL) with informational independence as first-class notion. Unlike in FOL, where quantified variables always functionally depend on all the previously quantified ones, the values for quantified variables in IF can be chosen independently of the values of specific variables quantified before in the formula. From a general game-theoretic viewpoint, however, the IF semantics exhibits some limitations. It treats the players asymmetrically, truly allowing only one of the two players to have imperfect information. In addition, sentences of the logic can only encode the existence of a uniform winning strategy for one of the two players and, as a consequence, IF does admit undetermined sentences, which are neither true nor false.

In this talk I will present an extension of IF, called *Alternating (In)Dependence Friendly Logic* (ADIF), tailored to overcome these limitations and that appears more adequate when reasoning about games with full imperfect information is the main concern. To this end, we introduce a novel compositional semantics, generalising Hodges' semantics for IF based on trumps/teams [3, 7, 6], which (i) allows for restricting the two players, aiming at describing both symmetric and asymmetric imperfect information games, (ii) recovers the law of excluded middle for sentences, and (iii) grants ADIF the full descriptive power of *Second Order Logic*. We also provide both an equivalent Herbrand-Skolem semantics and a game-theoretic semantics for the prenex fragment of ADIF, the latter being defined in terms of a determined infinite-duration game that precisely captures the compositional semantics on finite structures.

This is joint work with Dylan Bellier, Massimo Benerecetti, and Dario Della Monica.

[1] J. Hintikka. *Logic, Language-Games and Information: Kantian Themes in the Philosophy of Logic.* Oxford University Press, 1973.

[2] J. Hintikka and G. Sandu. Informational Independence as a Semantical Phenomenon. In *ICLMPS'89*, pages 571–589. Elsevier, 1989.

[3] W. Hodges. Compositional Semantics for a Language of Imperfect Information. *LJIGPL*, 5(4):539–563, 1997.

[4] K. Lorenz. Dialogspiele als Semantische Grundlage von Logikkalkülen. *AMLG*, 11:32–55, 1968.

[5] P. Lorenzen. Ein Dialogisches Konstruktivitätskriterium. In *SFM'59*, pages 193–200. PWN, 1961.

[6] A.L. Mann, G. Sandu, and M. Sevenster. *Independence-Friendly Logic - A Game-Theoretic Approach.* CUP, 2011.

[7] J.A. Väänänen. *Dependence Logic: A New Approach to Independence Friendly Logic.*, volume 70 of *London Mathematical Society Student Texts.* CUP, 2007.

[8] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior.* Princeton University Press, 1944.

► JOANNA OCHREMIAK, *On the Power of Symmetric Linear Programs.*
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We consider families of symmetric linear programs (LPs) that decide a property of graphs (or other relational structures) in the sense that, for each size of graph, there is an LP defining a polyhedral lift that separates the integer points corresponding to graphs with the property from those corresponding to graphs without the property. We show that this is equivalent, with at most polynomial blow-up in size, to families of symmetric Boolean circuits with threshold gates.

When we consider polynomial-size LPs, the model is equivalent to definability in a non-uniform version of fixed-point logic with counting (FPC). Known upper and lower bounds for FPC apply to the non-uniform version. In particular, this implies that the class of graphs with perfect matchings has polynomial-size symmetric LPs, while we obtain an exponential lower bound for symmetric LPs for the class of Hamiltonian graphs.

The talki is based on joint work with Albert Atserias and Anuj Dawar [1].

[1] Albert Atserias, Anuj Dawar, and Joanna Ochremiak. On the power of symmetric linear programs. *J. ACM*, 68(4), jul 2021.

▸ REVANTHA RAMANAYAKE, *Sequent calculi with restricted cuts for non-classical logics.*

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The primary motivation for cut-elimination is that it leads to a proof calculus with the subformula property. Such a proof calculus has a restricted proof search space and this is a powerful aid for investigating the properties of the logic. Unfortunately, many substructural and modal logics of interest lack a sequent calculus that supports cut-elimination. The overwhelming response since the 1960s has been to generalise the sequent calculus in a bid to regain cut-elimination. The price is that these generalised formalisms are more complicated to reason about and implement.

There is an alternative: remain with the sequent calculus by accepting weaker (but still meaningful) versions of the subformula property. We will discuss how cut-free hypersequent proofs can be transformed into sequent calculus proofs in a controlled way [1]. Combined with the quite general methodology [2] for transforming Hilbert axiomatic extensions into cut-free hypersequent calculi, this leads to an algorithm taking a Hilbert axiomatic extension to a sequent calculus with a weak subformula property.

Can we avoid this detour through the hypersequent calculus? This goes to the heart of a new programme called *cut-restriction* that aims to adapt Gentzen's celebrated cut-elimination argument systematically so that cut-formulas are restricted (when elimination is not possible). We will present the early results in this programme: from arbitrary cuts to analytic cuts in the sequent calculi for bi-intuitionistic logic and $S5$ via a uniform cut-restriction argument (the results themselves are well-known).

Based on joint work with Agata Ciabattoni (TU Wien) and Timo Lang (UCL).

[1] Agata Ciabattoni and Timo Lang and Revantha Ramanayake, *Bounded-analytic Sequent Calculi and Embeddings for Hypersequent Logics*, ***Journal of Symbolic Logic***, vol. 86 (2021), no. 2, pp. 635–668.

[2] A. Ciabattoni and N. Galatos and K. Terui, *From axioms to analytic rules in nonclassical logics*, ***Proceedings of the Twenty-Third Annual IEEE Symposium on Logic in Computer Science (LICS)*** Pittsburgh, PA, USA, IEEE Computer Society, 2008, pp. 229–240.

▸ ALEXIS SAURIN, *On the dynamics of cut-elimination for circular and non-wellfounded proofs.*

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In this talk, I will consider the structural proof theory of fixed-point logics and their cut-elimination theorems, focusing on their computational content.

More specifically, I will consider logics with least and greatest fixed-points, expressing inductive and coinductive properties, and proof systems for those logics admitting "circular" and non-wellfounded proofs [1, 2, 4, 5]. Those derivations are finitely branching but admit infinitely deep branches, possibly subject to some regularity conditions. Circular derivations are closely related with proofs by infinite descent [3] and shall be equipped with a global condition preventing vicious circles in proofs.

In order to unveil the computational content of those logical systems, I will concentrate on linear logic extended with least and greatest fixed points (μLL), that is, on the μ-calculus considered in a linear setting, where the structural rules of contraction and weakening are prohibited (or carefully controlled at least). In particular, following the spirit of structural proof-theory and of the Curry-Howard correspondence, we will be interested not only in the structure of provability but also in the structure of proofs themselves, corresponding to programs (while formulas correspond to data and codata types).

I will first introduce the non-wellfounded proof systems for μLL and for its exponential-free fragment, μMALL (that is, multiplicative and additive linear logic with least and greatest fixed points). After establishing cut-elimination for μMALL [2], I will show how to generalize the cut-elimination result to μLL (as well as to the intuitionistic and classical non-wellfounded sequent calculi). After that, I will discuss limitations of the validity condition considered above, from a computational perspective, and introduce a more flexible validity condition, called bouncing-validity [1], and establish a cut-elimination theorem for this richer system which, while proving the same theorems, admits more valid proofs that is, through the bridge of the Curry-Howard correspondence, more programs.

[1] David Baelde, Amina Doumane, Denis Kuperberg, and Alexis Saurin, *Bouncing Threads for Circular and Non-wellfounded Proofs – Towards Compositionality with Circular Proofs*, ***To appear in 37th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2022*** (Haifa, Israel), 2022.

[2] David Baelde, Amina Doumane, and Alexis Saurin, *Infinitary Proof Theory: the Multiplicative Additive Case*, ***In 25th EACSL Annual Conference on Computer Science Logic, CSL 2016*** (Marseille, France), (LIPIcs), Vol. 62. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 42:1–42:17.

[3] James Brotherston and Alex Simpson, *Sequent Calculi for Induction and Infinite Descent*, ***Journal of Logic and Computation***, vol. 21 (2011), no. 6, pp. 1177–1216.

[4] Jérôme Fortier and Luigi Santocanale, *Cuts for Circular Proofs: Semantics and Cut-elimination*, ***Computer Science Logic 2013 (CSL 2013), CSL 2013*** (Torino, Italy), (Simona Ronchi Della Rocca, editor), (LIPIcs), , Vol. 23. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2013, 248–262.

[5] Luigi Santocanale, *A Calculus of Circular Proofs and Its Categorical Semantics*, ***Foundations of Software Science and Computation Structures***, (Mogens Nielsen and Uffe Engberg, editors), vol. 2303, Lecture Notes in Computer Science, Springer, 2002, pp. 357–371

Abstracts of invited talks in the Special Session on Model Theory

▸ SYLVY ANSCOMBE, *Henselian discretely valued fields and existential AKE principles.*
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Ax–Kochen/Ershov (AKE) principles are known for various classes of henselian valued fields, including tame valued fields (itself including the case of equal characteristic zero) and the unramified mixed characteristic case. While the case of equal characteristic $p > 0$ remains mysterious, in full generality, there has been progress in understanding the existential fragment of theories of such henselian valued fields.

In 2003, Denef and Schoutens obtained an axiomatization (and decidability) of the existential theory of $\mathbb{F}_p((t))$, expanded by a parameter for t, assuming Resolution of

Singularties in characteristic $p > 0$. Recently, with Dittmann and Fehm, we have shown a similar result with a weaker assumption. More generally: assuming a weak consequence of resolution of singularities, we obtain a transfer principle for the existential decidability of fields equipped with a discrete equicharacteristic henselian valuation and a distinguished uniformizer.

▶ SAMUEL BRAUNFELD AND MICHAEL C. LASKOWSKI, *Monadic dividing lines and hereditary classes.*

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A theory T is monadically NIP if every expansion of T by unary predicates is NIP. We will discuss how monadic NIP manifests in the theory T itself rather than just in unary expansions, and how this can be used to produce structure or non-structure in hereditary classes. Analogous results concerning monadic stability may also be discussed.

[1] SAMUEL BRAUNFELD AND MICHAEL C. LASKOWSKI, *Characterizations of monadic NIP*, ***Transactions of the American Mathematical Society, Series B***, vol. 8 (2021), pp. 948–970.

▶ JAN DOBROWOLSKI, *Tameness in positive logic.*

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Positive logic is a very flexible framework unifying full first-order logic with several other settings, such as Robinson's logic (which studies existentially closed models of a possibly non-companionable first-order universal theory), hyperimaginary extensions of first-order theories (which are obtained by adding quotients by type-definable equivalence relations), and, in certain aspects, continuous logic.

The study of tameness in those contexts goes back to A. Pillay's work on simple Robinson's theories ([3]), and I. Ben Yaacov's work on simple compact abstract theories ([1]). In the talk, I will present a joint work with M. Kamsma on NSOP_1 in positive logic and a joint work in progress with R. Mennuni on NIP in positive logic, discussing in particular the main motivating examples for the two projects: existentially closed exponential fields (studied before by L. Haykazyan and J. Kirby in [2]) and existentially closed ordered abelian groups with an automorphism.

[1] I. Ben Yaacov. *Simplicity in compact abstract theories*, Journal of Mathematical Logic, 03(02):163–191, 2003.

[2] L. Haykazyan, J. Kirby, *Existentially closed exponential fields*, Israel Journal of Mathematics, 241(1):89–117, 2021.

[3] A. Pillay. *Forking in the Category of Existentially Closed Structures*, Quaderni di Matematica, 6:23–42, 2000.

▶ ALEX KRUCKMAN, *Kim's lemmas and tree properties.*

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One of the most important technical steps in the development of simplicity theory in the 1990s was a result now known as Kim's Lemma: In a simple theory, if a formula $\varphi(x;b)$ divides over a model M, then $\varphi(x;b)$ divides along every Morley sequence in $\mathrm{tp}(b/M)$. More recently, variants of Kim's Lemma have been shown by Chernikov, Kaplan, and Ramsey to follow from, and in fact characterize, two generalizations of simplicity in different directions: the combinatorial dividing lines NTP_2 and NSOP_1. After surveying the Kim's Lemmas of the past, I will suggest a new variant of Kim's Lemma, and a corresponding new model-theoretic tree property, which generalizes both TP_2 and SOP_1. I will also compare this new tree property with the Antichain Tree Property (ATP), another tree property generalizing both TP_2 and SOP_1, which was introduced recently by Ahn and Kim. This is joint work with Nick Ramsey.

▸ MARIANA VICARÍA, *Elimination of imaginaries in* $\mathbb{C}((t^\Gamma))$.
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One of the most striking results in the model theory of henselian valued fields is the Ax-Kochen theorem, which roughly states that the first order theory of a finitely ramified henselian valued field is completely determined by the first order theory of the residue field and its value group.

A model theoretic principle follows from this theorem: any model theoretic question about the valued field can be reduced into a question to its residue field, its value groups and their interaction in the field.

Our leading question is: Can one obtain an Ax-Kochen style theorem to eliminate imaginaries in henselian valued fields?

Following the Ax-Kochen principle, it seems natural to look at the problem in two orthogonal directions: one can either make the residue field tame and understand the problems that the value group brings naturally to the picture, or one can assume the value group to be very tame and study the issues that the residue field would contribute to the problem. In this talk we will address the first approach. I will explain the sorts required to obtain elimination of imaginaries in henselian valued fields of equicharacteristic zero with residue field algebraically closed and more general value groups.

▸ TINGXIANG ZOU, *The Elekes-Szabó problem for cubic surfaces.*
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The Elekes-Szabó problem asks when a complex variety $V \subseteq \prod_{i=1}^{3} W_i$ has unexpected large intersections with Cartesian products of finite subsets $X_i \subseteq W_i$ for $1 \leq i \leq 3$. Under the assumption that X_i's are in *general position*, Elekes and Szabó proved that one can always find commutative algebraic groups in this scenario. We explored the case when W_i's are a fixed cubic surface S in $\mathbb{P}^3(\mathbb{C})$ and V is the collinearity relation with the assumption that X_i does not concentrate on any one-dimensional subvarieties of S, which substantially weakens the general position assumption. We proved that when S is a smooth quadric surface union a plane, then one cannot find such X_i's. When S is an irreducible smooth cubic surface, then X_i's would contain a union of translates of arithmetic progressions on the family of planar cubic curves of S. But the existence of such X_i's is still open. This is a work-in-progress joint with Martin Bays and Jan Dobrowolski.

Abstracts of invited talks in the Special Session on Philosophy of Mathematics

▶ TIM BUTTON, *MOON theory: Mathematical Objects with Ontological Neutrality.*
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The iterative notion of set starts with a simple, coherent story, and yields a paradise of mathematical objects, which "provides a court of final appeal for questions of mathematical existence and proof" ([5, p.26]). But it does not present an attractive mathematical ontology: it seems daft to say that every mathematical object is "really" some (pure) set. My goal, in this paper, is to explain how we can inhabit the set-theorist's paradise of *mathematical objects* whilst remaining *ontologically neutral*.

I start by considering stories with this shape: (1) Gizmos are found in stages; every gizmo is found at some stage. (2) Each gizmo reifies (some fixed number of) relations (or functions) which are defined only over earlier-found gizmos. (3) Every gizmo has (exactly one) colour; same-coloured gizmos reify relations in the same way; same-coloured gizmos are identical iff they reify the same relations.

Such a story can be told about (iterative) sets: they are monochromatic gizmos which reify one-place properties. But we can also tell such stories about gizmos other than sets. By tidying up the general idea of such stories, I arrive at the notion of a MOON theory (for Mathematical Objects with Ontological Neutrality).

With weak assumptions, I obtain a metatheorem: *all MOON theories are synonymous.* Consequently, they are (all) synonymous with a theory which articulates the iterative notion of set (LT_+; see [1]). So: all MOON theories (can) deliver the set-theorist's paradise of mathematical objects. But, since different MOON theories have different (apparent) ontologies, we attain ontological neutrality.

My metatheorem generalizes some of my work on Level Theory ([1], [2], [3]). It also delivers a partial realization of Conway's "Mathematician's Liberation Movement" [4, p.66].

[1] Button, T. Level Theory, Part 1: Axiomatizing the bare idea of a cumulative hierarchy of sets. *Bulletin Of Symbolic Logic.* **27**, 436-60 (2021)

[2] Button, T. Level Theory, Part 2: Axiomatizing the bare idea of a potential hierarchy. *Bulletin Of Symbolic Logic.* **27**, 461-84 (2021)

[3] Button, T. Level Theory, Part 3: A boolean algebra of sets arranged in well-ordered levels. *Bulletin Of Symbolic Logic.* **28**, 1-26 (2022)

[4] Conway, J. On Numbers and Games. (Academic Press, Inc,1976)

[5] Maddy, P. Naturalism in Mathematics. (Oxford University Press,1997)

▶ LAURA CROSILLA, *Hermann Weyl and the roots of mathematical logic.*
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Hermann Weyl's book *Das Kontinuum* [2] presents a coherent and sophisticated approach to analysis from a predicativist perspective. In the first chapter of [2], Weyl introduces a system of predicative sets, built "from the bottom up" starting from the natural numbers. He then goes on to show that large portions of 19th century analysis can be developed on that predicative basis. *Das Kontinuum* anticipated and inspired fundamental ideas in mathematical logic, ideas that we find in the logical analysis of predicativity of the 1950-60's, in Solomon Feferman's work on predicativity and in Errett Bishop's constructive mathematics. The seeds of *Das Kontinuum* are already visible in the early [1], where Weyl, among other things, offers a clarification of Zermelo's axiom schema of Separation. In this talk, I examine key intriguing ideas in [1], ideas that witness important debates among mathematicians at the beginning of the 20th century. I then argue that aspects of [1] foreshadow fundamental features of *Das Kontinuum*. This allows us to consider [2] under the new light offered by [1].

[1] Weyl, H., 1910, *Über die Definitionen der mathematischen Grundbegriffe*, Mathematisch-naturwissenschaftliche Blätter, 7, pp. 93–95 and pp. 109–113.

[2] Weyl, H., 1918, *Das Kontinuum. Kritische Untersuchungen über die Grundlagen der Analysis*, Veit, Leipzig. Translated in English, Dover Books on Mathematics, 2003. (Page references are to the translation).

▸ SALVATORE FLORIO, *Conceptions of absolute generality.*
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What is absolutely unrestricted quantification? Recent work on the possibility of absolute generality has highlighted that there are different legitimate answers to this question. Relying especially on [1], [2], and [3], I explore some of these answers, and their relations, in the context of different forms of type theory. The result is an initial analysis of different conceptions of absolute generality and of the theoretical value of the corresponding kinds of generalization.

[1] Salvatore Florio and Nicholas K. Jones, *Unrestricted quantification and the structure of type theory*, ***Philosophy and Phenomenological Research***, vol. 102 (2021), no. 1, pp. 44–64.

[2] Tim Button and Robert Trueman, *Against cumulative type theory*, ***The Review of Symbolic Logic***, forthcoming.

[3] Salvatore Florio and Nicholas K. Jones, *Two conceptions of absolute generality*, manuscript.

▸ BRICE HALIMI, *Geometrizing Kripke modal semantics.*
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Kripke semantics for propositional modal logic is based on the notion of accessibility between possible worlds. The purpose of my talk is to take the latter notion literally, i.e., as indicating the existence of a path between two worlds, and thus to geometrize Kripke semantics by considering the space underlying the collection of all possible worlds as an important semantical feature in its own right. The resulting new modal semantics is worked out in a setting coming from Riemannian geometry, where Kripke semantics is shown to correspond to a special case (namely, the discrete one), and thus geometrization to amount to a generalization. Several completeness results, established between variants of well-known modal systems and certain geometric-metric properties, illustrate the import of the new framework.

Abstracts of invited talks in the Special Session on Proof Theory and Ordinal Analysis

▸ BAHAREH AFSHARI, *From interpolation to proofs.*
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From a proof-theoretic perspective, the idea that interpolation is tied to provability is a natural one. Thinking about Craig interpolation, if a 'nice' proof of a valid implication

$\phi \to \psi$ is available, one may succeed in defining an interpolant by induction on the proof-tree, starting from leaves and proceeding to the implication at the root. This method has recently been applied even to fixed point logics admitting cyclic proofs [1, 4]. In contrast, for uniform interpolation, there is no single proof to work from but a collection of proofs to accommodate: a witness to each valid implication $\phi \to \psi$ where the vocabulary of ψ is constrained. Working over a set of prospective proofs and relying on the structural properties of sequent calculus is the essence of Pitts' seminal result on uniform interpolation for intuitionistic logic [3].

In this talk we explore the opposite direction of the above endeavour, arguing that uniform interpolation can entail completeness of a proof system. We will demonstrate this in the case of propositional modal μ-calculus by showing that the uniform interpolants obtained from cyclic proofs [2] play an important role in establishing completeness for the natural Hilbert axiomatisation of this fixed point logic.

[1] Bahareh Afshari and Graham E. Leigh, *Lyndon interpolation for modal mu-calculus*, ***Language, Logic, and Computation TbiLLC 2019*** (Cham), (Aybüke Özgün and Yulia Zinova, editors), vol. 13206, Lecture Notes in Computer Science, 2022, pp. 197–213.

[2] Bahareh Afshari, Graham E. Leigh and Guillermo Menédez Turata, *Uniform interpolation from cyclic proofs: The case of modal mu-calculus.*, ***Automated Reasoning with Analytic Tableaux and Related Methods - 30th International Conference, TABLEAUX 2021*** (Birmingham, UK), (Anupam Das and Sara Negri, editors), vol. 12842, Springer, 2021, pp. 335–353.

[3] Andrew M. Pitts, *On an interpretation of second order quantification in first order intuitionistic propositional logic*, ***Journal of Symbolic Logic***, vol. 57 (1992), no. 1, pp. 33–52.

[4] Daniyar Shamkanov, ***Circular Proofs for Gödel-Löb Logic***, arXiv preprint arXiv:1401.4002, 2014.

► JUAN P. AGUILERA, *The Π^1_2-spectrum conjecture.*
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The Π^1_2-soundness ordinal of a theory T, denoted $o^1_2(T)$, is a measure of how close T is to being Π^1_2-correct. The Π^1_2-spectrum conjecture asserts that the possible values of $o^1_2(T)$ for recursively enumerable extensions of ACA_0 are precisely the Σ^1_1-definable epsilon numbers. In this talk, we present a proof of the following theorem, which is formalizable in weak set theories: If the Π^1_2-Spectrum Conjecture fails, then Second-Order Arithmetic is consistent. This is joint work with Fedor Pakhomov.

► DAVID FERNÁNDEZ-DUQUE, *Noetherian Gödel Logics.*
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Noetherian Gödel logics are many-valued logics where the set of truth values is a closed subset of $[0, 1]$ without infinite ascending sequences. These logics are parametrized by countable ordinals, so that $\mathsf{G}^{\downarrow}_{\alpha}$ is the logic with truth values inversely isomorphic to $\alpha + 1$. In this talk we discuss the complexity of satisfiability and validity for each Noetherian Gödel logic, strengthening and generalizing results of Baaz-Leitsch-Zach and Hájek. Specifically, we show that the complexity of satisfiability and validity in $\mathsf{G}^{\downarrow}_{\alpha}$ are related to Σ^1_1 and Π^1_1 formulas, respectively, over $(\mathbb{L}_\beta)_{\beta\leq\alpha}$.

This is joint work with Juan P. Aguilera and Jan Bydzovsky

▶ GERHARD JÄGER, *The admissible extension of subsystems of second order arithmetic.*
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Given a first order structure $\mathfrak{M}$, the next admissible $\mathbb{HYP}_{\mathfrak{M}}$ and Barwise's cover $\mathbb{C}\mathrm{ov}_{\mathfrak{M}}$ – provided that $\mathfrak{M}$ is a model of Kripke-Platek set theory KP – are examples of structures that extend $\mathfrak{M}$ to a (in some sense) larger admissible set; see his textbook "Admissible Sets and Structures". But observe that these processes do not affect the underlying $\mathfrak{M}$.

Now let T be a a subsystem of second order arithmetic. What happens when we combine T with Kripke-Platek set theory KP? Let us start off from a structure $\mathfrak{M} = (\mathbb{N}, \mathbb{S}, \in)$ of the natural numbers $\mathbb{N}$ and collection of sets of natural numbers $\mathbb{S}$ that has to obey the axioms of T. Then we erect a set-theoretic world with transfinite levels on top of $\mathfrak{M}$ governed by the axioms of KP. However, owing to the interplay of T and KP, either theory's axioms may force new sets of natural to exists which in turn may engender yet new sets of naturals on account of the axioms of the other. Therefore, the admissible extension of T is usually not a conservative extension of T.

It turns out that for many familiar theories T, the second order part of the admissible extension of T equates to T augmented by transfinite induction over all initial segments of the Bachmann-Howard ordinal.

This is joint work with Michael Rathjen.

▶ RICHARD MATTHEWS, *On the Constructive Constructible Universe.*
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Gödel's Constructible Universe, L, was introduced to show the consistency of the Axiom of Choice and the Generalized Continuum Hypothesis with the axioms of ZF and is the smallest inner model of ZF. That is, it is the minimal submodel that satisfies ZF and contains all the ordinals of the background universe. In this talk we shall see how vastly different the situation can be in constructive set theories.

Firstly, we shall investigate L over CZF. Via a proof-theoretic ordinal analysis of Power Kripke Platek combined with realizability, we shall show that it is not possible to prove that Exponentiation holds in L. Therefore, over CZF, the Constructible Universe may fail to be an inner model of full CZF. Secondly, we shall explore the concept of an ordinal in the constructive setting. We shall see that, without the law of excluded middle, ordinals need not satisfy many of the standard, expected properties and instead can have very strange behaviour. In particular, over IZF, we shall see that it is possible for there to be an ordinal which is not in the constructible universe, answering a question of Lubarsky. This is joint work with Michael Rathjen.

▶ GUNNAR WILKEN, *Isominimal realizations of patterns.*
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Elementary patterns of resemblance are ordinal notations that can be approached from both a combinatorial and a semantic angle. The former derives from the observation that patterns are so-called respecting forests, while the latter is tied to the existence of unique isominimal realizations in the ordinals when interpreting the edges of patterns by elementary substructurehood. For patterns of order 1 the characterization of isominimal realizations is quite perspicuous, whereas patterns of order 2 already pose challenges, some of which I will address in my talk.

Abstracts of invited talks in the Special Session on Reverse Mathematics and Combinatorial Principles

► CHRIS CONIDIS, *The computability of the Artin-Rees Lemma and Krull Intersection Theorem.*
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We will examine the proofs of two related algebraic theorems, namely the Artin-Rees Lemma (AR) and the Krull Intersection Theorem (KIT). These related arguments appear in many Algebra textbooks in which AR is used to prove KIT. First, we will show that AR and KIT each follow from weak König's Lemma (WKL_0). We will then go on to show that, in the context of infinite sequences of rings, the uniform Artin-Rees Lemma (UAR) still follows from WKL_0, but the uniform Krull Intersection Theorem (UKIT) does not.

[1] H. Matsumura, ***Commutative Ring Theory***, Cambridge University Press, 2006.

► DENIS R. HIRSCHFELDT, *The strength of versions of Mycielski's Theorem.*
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Mycielski's Theorem is a Ramsey-theoretic result on the reals with versions for measure and for category. These imply respectively that there is a perfect tree whose paths are all relatively 1-random, and that there is a perfect tree whose paths are all relatively 1-generic. In fact, in relativized form, the latter two statements are equivalent to the two versions of Mycielski's Theorem. I will discuss joint work with Carl G. Jockusch, Jr. and Paul E. Schupp on the computability-theoretic and reverse-mathematical strength of these statements.

► KATARZYNA W. KOWALIK, *A non speed-up result for the chain-antichain principle over a weak base theory.*
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The chain-antichain principle (CAC), a well-known consequence of Ramsey's Theorem for pairs and two colours, says that for every partial order on $\mathbb{N}$ there exists an infinite chain or antichain with respect to this order. We study the strength of this principle over the weak base theory RCA_0^*, which is obtained from RCA_0 by replacing the Σ^0_1-induction scheme with Δ^0_1-induction.

It was shown by Patey and Yokoyama in [3] that RT^2_2 is Π^0_3-conservative over RCA_0 and from [4] it follows that RT^2_2 is also Π^0_3-conservative over RCA_0^* (cf. [1]). The conservativity results lead to the question whether RT^2_2 has significantly shorter proofs for Π^0_3-sentences. The answer depends on the choice of the base theory: it was proved in [2] that RT^2_2 can be polynomially simulated by RCA_0 for Π^0_3-sentences but it has non-elementary speed-up over RCA_0^* for Σ^0_1-sentences.

The speed-up result was obtained by the use of the exponential lower bound for the finite version of RT^2_2. However, it follows from Dilworth's theorem that the upper bound for the finite version of CAC is polynomial. This suggests that CAC, despite being a relatively strong consequence of RT^2_2, might not have an analogous speed-up over RCA_0^*. We confirm this conjecture by constructing a two-step forcing interpretation of

RCA_0^*+CAC in RCA_0^*.

[1] Leszek A. Kołodziejczyk, Katarzyna W. Kowalik, Keita Yokoyama, *How strong is Ramsey's theorem if infinity can be weak?* Submitted. Available at arXiv:2011.02550.

[2] Leszek A. Kołodziejczyk, Tin Lok Wong, Keita Yokoyama, *Ramsey's theorem for pairs, collection, and proof size.* Submitted. Available at arXiv:2005.06854.

[3] Ludovic Patey, Keita Yokoyama, *The proof-theoretic strength of Ramsey's theorem for pairs and two colors*, ***Advances in Mathematics***, vol. 330 (2018), pp. 1034–1070.

[4] Keita Yokoyama, *On the strength of Ramsey's theorem without Σ_1-induction*, ***Mathematical Logic Quarterly***, vol. 59 (2013), no. 1-2, pp. 108–111.

▸ GIOVANNI SOLDÀ, *On the strength of some first-order problems corresponding to Ramseyan principles.*

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Given a represented space X, we say that a problem f with $\mathrm{dom}(f) \subseteq X$ is *first-order* if its codomain is $\mathbb{N}$. In this talk, we will study, from the point of view of Weihrauch reducibility, some first-order problems corresponding to Ramseyan combinatorial principles.

We will start by analyzing some problems that can be seen naturally as first-order: more specifically, after mentioning some well-established results due to Brattka and Rakotoniaina [1], we will proceed to study some principles whose strengths, form a reverse mathematical perspective, lie around $\mathsf{I}\Sigma^0_2$, as proved mainly in [2].

We will then move to study the *first-order part* ${}^1f$ of problems f which cannot be presented as first-order ones: intuitively speaking, ${}^1f$ corresponds the strongest first-order problem Weihrauch reducible to f. The first-order part operator was introduced by Dzhafarov, Solomon and Yokoyama in unpublished work, and it has already proved to be a valuable tool to gauge the strengths of various problems according to Weihrauch reducibility. After giving some technical results on this operator, we will focus on ${}^1(\mathsf{RT}^2_2)$, presenting various results on the position of its degree in the Weihrauch lattice.

The results presented are joint work with Arno Pauly, Pierre Pradic, and Manlio Valenti.

[1] Vasco Brattka and Tahina Rakotoniaina, *On the uniform computational content of Ramsey's theorem*, ***The Journal of Symbolic Logic***, vol. 82 (2015), no. 4, pp. 1278–1316

[2] Leszek A. Kolodziejczyk, Henryk Michalewski, Pierre Pradic, and Michał Skrzypczak, *The logical strength of Büchi's decidability theorem*, ***25th EACSL Annual Conference on Computer Science Logic (CSL 2016)*** (Marseille, France), (Jean-Marc Talbot and Laurent Regnier), vol. 62, Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2016, pp. 36:1–36:16.

▸ WEI WANG, *Ackermann Function and Reverse Mathematics.*

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In 1928, Ackermann [1] defined one of the first examples of recursive but not primitive recursive functions. Later in 1935, Rózsa Péter [5] provided a simplification, which is now known as Ackermann or Ackermann-Péter function. The totality of Ackermann-Péter function is an interesting subject in the study of fragments of first order arithmetic. Kreuzer and Yokoyama [4] prove that the totality of Ackermann-Péter function is equivalent to a Σ_3-proposition called $P\Sigma_1$. And $P\Sigma_1$ has played important roles in reverse mathematics in recent years. We will see some examples in this talk, including some joint works [2, 3] of the speaker and logicians in Singapore.

[1] Ackermann, Wilhelm, *Zum Hilbertschen Aufbau der reellen Zahlen*, ***Mathematische Annalen***, 99(1):118–133, 1928.

[2] Chong, Chitat and Li, Wei and Wang, Wei and Yang, Yue, *On the strength of Ramsey's theorem for trees*, ***Advances in Mathematics***, 369:107180, 39 pp, 2020.

[3] Chong, Chitat and Wang, Wei and Yang, Yue, *Conservation Strength of The Infinite Pigeonhole Principle for Trees*, ***Israel Journal of Mathematics***, to appear, *https://arxiv.org/abs/2110.06026.*

[4] Kreuzer, Alexander P. and Yokoyama, Keita, *On principles between Σ_1- and Σ_2-induction, and monotone enumerations*, ***Journal of Mathematical Logic***, 16(1):1650004, 21 pp, 2016.

[5] Péter, Rózsa, *Konstruktion nichtrekursiver Funktionen*, ***Mathematische Annalen***, 111(1):42–60, 1935.

► LIAO YUKE, *Recursive coloring without Δ^0_3 witness for Hindman theorem.*
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We give an example of a recursive coloring of integers which has no Δ^0_3 witness for Hindman theorem and an example of a recursive coloring of integers such that any Π^0_3 set of integers whose any two elements are apartness is not a witness for Hindman theorem.

[1] Andreas R. Blass,Jeffry L. Hirst,Stephen G. Simpson, *Logical analysis of some theorems of combinatorics and topological dynamics*, ***Contemporary Mathematics***, vol. 65 (1987), pp. 125–156.

Abstracts of invited talks in the Special Session on Set Theory

► BEN DE BONDT, *Some remarks on Namba-type forcings.*
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For the purpose of this abstract, let "a Namba-type forcing" be any forcing that forces ω_2 to get cofinality ω and doesn't collapse ω_1. It is well known that the existence of a semiproper Namba-type forcing is equivalent to a Strong Chang's Conjecture, but that instead the existence of a stationary set preserving Namba-type forcing is provable in plain ZFC. However, in the context of questions on iterated-forcing-using-side-conditions, it is natural to ask whether one can demand more than mere stationary set preservation and get provably in ZFC a Namba-type forcing that allows many (but not necessarily club many) models for which there exist sufficient semi-generic conditions. In this talk I will discuss a "side-condition version" $\mathbb{P}$ of Namba forcing and explain that there exists a very natural projective stationary family of countable elementary submodels of H_θ such that $\mathbb{P}$ is semiproper *with respect to these models*. In fact, we can consider a notion of strong semiproperness, in analogy to the notion of strong properness and verify that $\mathbb{P}$ satisfies it, again with respect to these distinguished models.

As an application of this approach towards Namba forcing, we discuss a particularly natural presentation of an "ersatz iterated Namba forcing" which, given an increasing sequence $(\kappa_\alpha : \alpha < \gamma)$ of regular cardinals $\geq \omega_2$, adds for every $\alpha < \gamma$ a countable cofinal subset of κ_α, while at the same time preserving stationarity of stationary subsets of ω_1. In the proof, we will make strong use of a technique involving labelled trees and games played on such trees that appears in [1].

Finally, we will mention closely related ongoing work and remaining questions. This talk is based on joint work with my thesis supervisor Boban Veličković.

[1] Matthew Foreman and Menachem Magidor, *Mutually stationary sequences of sets and the non-saturation of the non-stationary ideal on $P_\kappa(\lambda)$*, ***Acta Mathematica***, vol. 186 (2001), no. 2, pp. 271–300.

[2] Saharon Shelah, ***Proper and Improper Forcing***, Perspectives in Logic, vol. 5, Cambridge University Press, 1998.

▸ DIANA CAROLINA MONTOYA, *Independence for uncountable cardinals.*
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In this talk, we will discuss the concept of maximal independent families for uncountable cardinals. First, we will mention a summary of results regarding the existence of such families in the case of an uncountable regular cardinal. Specifically, we will focus on joint work with Vera Fischer regarding the existence of an indestructible maximal independent family, which turns out to be indestructible after forcing with generalized Sacks forcing.

In the second part, we will focus on the singular case and present two results obtained in joint work with Omer Ben-Neria. Finally, I will mention some open questions and future paths of research.

▸ THOMAS GILTON, MAXWELL LEVINE, AND ŠÁRKA STEJSKALOVÁ, *Club stationary reflection and consequences of square principles.*
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The square principle $\square_\mu$, for a cardinal μ, exerts a tremendous influence on the combinatorics of μ^+ implying, for example, that on μ^+ stationary reflection and the tree property fail, but that the approachability property holds. In [3], the authors showed that these three consequences of $\square_\mu$ are mutually independent, in the sense that any of their eight Boolean combinations are consistent, from large cardinals, at κ^{++}, where κ is either singular or regular.

Recently Levine, Stejskalová, and I ([1]) have continued this line of research, showing how to obtain *Club Stationary Reflection* together with a variety of other combinatorics at a double successor of a regular. Moreover, Stejskalová and I have recently shown ([2]) how to fold Prikry-type forcings into these arguments to obtain similar results at the double successor of a cofinality ω singular.

In this talk, we will briefly review the impact that $\square_\mu$ has on the combinatorics at μ^+, and then sketch the main ideas for a number of our theorems, both in the regular and singular cases. In particular, we will discuss how we use weakly compact Laver diamonds to build our focings, and we will discuss new preservation theorems for club stationary reflection. If time permits, we will also discuss current work which involves Magidor forcing and uncountable cofinality singulars.

[1] Thomas Gilton, Maxwell Levine, and Šárka Stejskalová. Trees and Stationary

Reflection at Double Successors of Regular Cardinals. Accepted to *The Journal of Symbolic Logic*

[2] Thomas Gilton and Šárka Stejskalová. Compactness Principles at $\aleph_{\omega+2}$. In preparation.

[3] James Cummings, Sy-David Friedman, Menachem Magidor, Assaf Rinot, and Dima Sinapova. The Eightfold Way. *The Journal of Symbolic Logic*. **83** (2018) no. 1, 349-371.

▸ SANDRA MÜLLER, *A stationary-tower-free proof of* Sealing *from a supercompact.*
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Sealing is a generic absoluteness principle for the theory of the universally Baire sets of reals introduced by Woodin. It is deeply connected to the Inner Model Program and plays a prominent role in recent advances in inner model theory. Woodin showed in his famous Sealing Theorem that in the presence of a proper class of Woodin cardinals Sealing holds after collapsing a supercompact cardinal. I will outline the importance of Sealing and discuss a new and stationary-tower-free proof of Woodin's Sealing Theorem that is based on Sargsyan's and Trang's proof of Sealing from iterability. This is joint work with Grigor Sargsyan and Bartosz Wcisło.

▸ DAMIAN SOBOTA, *P-measures in random extensions.*
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Let μ be a finitely additive probability measure on ω which vanishes on points, that is, $\mu(\{n\}) = 0$ for every $n \in \omega$. It follows immediately that μ is not σ-additive, however it may be almost σ-additive in the following weak sense. We say that μ is a *P-measure* if for every decreasing sequence (A_n) of subsets of ω there is a subset A such that $A \setminus A_n$ is finite for every n and $\mu(A) = \lim_n \mu(A_n)$. P-measures can be thought of as generalizations of P-points and similarly as in the case of P-points the existence of P-measures is independent of ZFC.

During my talk I will discuss basic properties of P-measures and show, at least briefly, that using old ideas of Solovay and Kunen one can obtain a non-atomic P-measure in the random model. The latter result implies that in this model ω^* contains a closed nowhere dense ccc P-set, which may be treated as a (weak) partial answer to the open question asking whether there are P-points in the random model.

This is a joint work with Piotr Borodulin-Nadzieja.

▸ JING ZHANG, *Making the diamond principle fail at an inaccessible cardinal.*
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It is a well-known theorem by Shelah that for any infinite cardinal $\lambda > \aleph_0$, $2^\lambda = \lambda^+$ is equivalent to $\diamondsuit(\lambda^+)$. However, the situation at inaccessible cardinals is different. Woodin produced a model where the diamond principle fails at a (greatly) Mahlo cardinal, based on the analysis of the Radin forcing. We will discuss the advantage and the limitation of such method. Furthermore, we demonstrate a new method giving rise to the failure of the diamond principle at an inaccessible cardinal, fundamentally different from Woodin's method. The differences from the previous method will be highlighted. Joint work with Omer Ben-Neria.

Abstract of Contributed Talks

▸ LUCA ACETO, ANTONIS ACHILLEOS, DUNCAN PAUL ATTARD, LÉO EXIBARD*, ADRIAN FRANCALANZA, KAROLIINA LEHTINEN, *Runtime monitoring for Hennessy-Milner logic with recursion over systems with data.*
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Runtime verification consists in checking whether a program satisfies a given specification by observing the trace it produces during its execution. In the regular setting, Hennessy-Milner logic with recursion (recHML), a variant of the modal μ-calculus, provides a versatile back-end for expressing linear- and branching-time specifications. In this paper, we study an extension of this logic [1] that allows to express properties over data values (i.e. values from an infinite domain) and examine which fragments can be verified at runtime. Data values are manipulated through first-order formulas over the underlying theory in modalities and through first-order quantification outside of them. They can also be stored using parameterised recursion variables.

Assuming decidability of the underlying first-order theory, we study how to generalise the classification known in the regular case. We further observe that restricting quantifier-free formulas in the modalities yields a logic that corresponds to register automata with non-deterministic reassignment, allowing us to ground our monitor synthesis algorithms, in the spirit of, and to derive impossibility results. In particular, contrary to the regular case, restricting to deterministic monitors strictly reduces the set of monitorable properties. We also note that further limiting quantifications to immediate bindings, we get $\mathsf{recHML}^{\mathsf{d}}$ [2], a logic previously introduced for monitoring events that carry data.

[1] Jan Friso Groote and Radu Mateescu, *Verification of Temporal Properties of Processes in a Setting with Data*, ***Proceedings of Algebraic Methodology and Software Technology, 7th International Conference, AMAST '98, vol. 1548 of* Lecture Notes in Computer Science*, pp. 74–90.***

[2] Luca Aceto, Ian Cassar, Adrian Francalanza and Anna Ingólfsdóttir, *On Runtime Enforcement via Suppressions*, ***Proceedings of the 29th International Conference on Concurrency Theory, CONCUR 2018, vol. 118 (34) of* LIPIcs*, pp. 1–17.***

▸ ANTONIS ACHILLEOS, ELENI BAKALI, AGGELIKI CHALKI*, ARIS PAGOURTZIS, *Descriptive complexity for hard counting problems with easy decision version.*
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The class #P is the class of functions that count the number of solutions to problems in NP. Since very few counting problems can be exactly computed in polynomial time (e.g. counting spanning trees), the interest of the community has turned to the complexity of approximating them. The class #PE of problems in #P with decision version in P is of great significance.

We focus on a subclass of #PE, namely TotP, the class of functions that count the total number of paths of NPTMs. TotP contains all self-reducible #PE functions and it is *robust*, in the sense that it has natural complete problems and it is closed under addition, multiplication and subtraction by one.

We present logical characterizations of TotP and two other *robust* subclasses of this class, building upon two seminal works about descriptive complexity for classes of counting problems [1, 2]. Specifically, to capture TotP, we use recursion on functions over second-order variables which, we believe, is of independent interest.

This work has been partially funded by the Basic Research Program PEVE 2020 of the National Technical University of Athens, and the project "MoVeMnt: Mode(l)s of Verification and Monitorability" (grant no 217987) of the Icelandic Research Fund.

[1] S. Saluja and K.V. Subrahmanyam and M.N. Thakur, *Descriptive Complexity of #P Functions*, ***Journal of Computer and System Sciences***, vol. 50, no. 3, pp. 493–505.

[2] M. Arenas and M. Muñoz and C. Riveros, *Descriptive Complexity for Counting Complexity Classes*, ***Logical Methods in Computer Science***, vol. 16, no. 1.

▸ MARK ADDIS, *Categorical representation of discrete dynamical systems computability.*
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In discrete dynamical systems computability is characterised by a state space of hereditarily finite sets combined with operations on those sets [4]. A class of states and operations transforms a given state into a succeeding one, and isomorphism and invariance relations between states define structural classes [3]. Such systems can be regarded as a generalisation of Gandy machines [2] thus enabling representation of computable processes which extend beyond Turing machines. Since the representation is complex the logical and philosophical gains achieved from simplifying it through the use of the abstract model theory approach of the theory of institutions [1] are considered. Such analysis contributes to the development of a category theory approach to the foundations of computability theory and philosophical reflection on the geometric aspects of certain kinds of computability.

[1] Diaconescu, R., *Three decades of institutions* ***Universal Logic: an Anthology***, (Beziau, J-Y, editor) Springer, Basel, 2012, pp. 309-322.

[2] Gandy, R., *Church's Thesis and principles for mechanisms*, ***The Kleene Symposium***, (Amsterdam, North Holland), (Barwise, J., Keisler, H., and Kunen, K., editors), 1980, pp. 123–148.

[3] Sieg, W., *Calculations by man and machine: mathematical presentation*, ***The Scope of Logic, Methodology and Philosophy of Science***, (Gärdenfors, P., Wolesnki, J., and Kijania-Placek, K. editors), Dordrecht, Kluwer, 2003, pp. 247–262.

[4] Sieg, W., *Church without dogma: axioms for computability*, ***New Computational Paradigms***, (Cooper, B., Löwe, B., and Sorbi, A., editors), Springer, New York, 2008, pp. 139–152.

▸ ISOLDE ADLER, BJARKI GEIR BENEDIKTSSON* AND DUGALD MACPHERSON, *Stability of generalized Johnson graphs.*
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Model-theoretic conditions such as stability have in recent years been shown to have important consequences for algorithms on graph classes (classes of – typically finite – graphs).

Here we consider the model theory of the Johnson graphs $J(n,k)$; here $n, k \in \mathbb{N}$, $1 < k < n-1$, and the vertices of $J(n,k)$ are the k-element subsets of an underlying set of size n, two vertices adjacent if their intersection has size $k-1$. These are well-known graphs which are distance-transitive, but they do not satisfy familiar graph-class tameness conditions such as having bounded clique-width.

We consider the common theory T_J of all Johnson graphs, and describe its completions and their models. In particular the limit theory $T_{J,\infty}$, where $k, n, n-k \to \infty$, is

complete. We give axioms for $T_{J,\infty}$, and show that it is ω-stable of Morley rank ω.
We finally give a complete description of all models of T_J.

▶ TIN ADLEŠIĆ AND VEDRAN ČAČIĆ*, *Formalizing assignment of types to terms in NFU*.
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Quine's *New Foundations for mathematical logic* from 1937 (upgraded later in 1951) was originally meant as a theory for rigorously formalizing *classes*: collections of objects satisfying certain predicates. In order to avoid the usual paradoxes, Quine stipulated that objects be *stratified*—divided according to types, so that a class is of a higher type than its elements. In search of consistency proof, proper classes were expelled (to metatheory) leaving only the theory of *sets*, the theory was reworded so stratification became *syntactic* (the assignment of natural numbers to variables in the formula expressing the defining predicate), and also *urelements* ("atoms" without elements, while not being equal to the empty class) were added. The resulting theory, NFU, was shown to be consistent by Jensen in 1969.

In a way, working in NFU seems a lot like working in any "usual" set theory—until it comes to the point where it's necessary to justify the existence of a certain set. And there are a lot of such situations: for example, proving a claim by mathematical induction amounts to showing that a certain set is inductive—but first we must ensure it is a set. Checking that a formula is stratified is a straightforward, if boring, task in the basic $\{\in, =\}$-language of set theory, but it becomes much more challenging when we add abstraction terms, functional and new relational symbols. To help us work, we have programmed a framework in Coq which can be used to establish whether the formula is stratified, find the *least* typization, and use that fact in establishing new notions.

▶ TIN ADLEŠIĆ* AND VEDRAN ČAČIĆ, *Tarski's theorem about choice and the alternative axiomatic extension of NFU*.
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The main advantage of NFU over plain NF is that it does not disprove the axiom of choice. Both the axiom of choice and the axiom of infinity are independent, but relatively consistent with NFU. Therefore, NFU + Inf + AC is a theory that is rich enough to encompass all the existent mathematics—but with some technical difficulties. Namely, it is hard to work with Kuratowski's ordered pairs because they are not type-leveled, meaning that the ordered pair does not have the same type as its projections. The fortunate circumstance is that everything can be developed irrespective of how we define ordered pairs, but making them type-leveled yields a significant simplification of exposition. A prevalent solution to that problem in contemporary literature is to postulate a new axiom, so-called *axiom of ordered pairs*. From our point of view, the introduction of that axiom lacks the proper motivation and justification for inclusion, and it also creates new problems; it can be expressed only by introducing new primitive notions. Such evasions are a common occurrence in contemporary NFU.

In theory NFU + Inf + AC we can define type-leveled ordered pairs using Tarski's theorem about choice which is equivalent to the AC. The main drawback for using NFU+Inf+AC is that we first need to develop all the necessary theory with Kuratowski's ordered pairs, prove the equivalence of the AC to Tarski's theorem, and only then define type-leveled ordered pairs. This is apparently unavoidable. However, we propose an approach which does that hard work only once: to start with NFU + Inf + Tarski, then define type-leveled ordered pairs, and then easily prove the equivalence of Tarski's theorem to the AC. In order to justify that shift of axioms, we must show that in NFU + Inf + AC we can prove the equivalence of the AC to the Tarski's theorem, but then it becomes a self-sufficient result one can just cite afterwards. It is also worth saying that Tarski seems much more justified as an axiom than the "axiom of ordered pairs". In order to complete our presentation, we also need to show that the same thing can be done in NFU + Inf + Tarski, but the equivalence proof can be mirrored by the former, and that proof will be in fact much simpler. In effect, those two theories are equiconsistent.

[1] G. Wagemakers, *New Foundations - A survey of Quine's set theory*, ***Instituut voor Tall, Logica en Informatie Publication Series***, X-89-02.

[2] J. B. Rosser, *Logic for mathematician*, ***Dover Publications***, 2008.

[3] S. Morris, *Quine, New Foundation, and the Philosophy of Set Theory*, ***Cambridge University Press***, 2018.

▸ JUAN PABLO AGUILERA, MARTÍN DIÉGUEZ, DAVID FERNÁNDEZ-DUQUE, AND BRETT MCLEAN*, *Gödel temporal logic*.

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We investigate a non-classical version of linear temporal logic (with next $\circ$, eventually $\Diamond$, and henceforth $\Box$ modalities) whose propositional fragment is Gödel–Dummett logic (which is well known both as a superintuitionistic logic and a t-norm fuzzy logic). The importance of both linear temporal logic and of fuzzy logics in computer science is well established.

We define the logic using two natural semantics—a real-valued semantics and a semantics where truth values are captured by a linear Kripke frame—and can show that these indeed define one and the same logic. Although this Gödel temporal logic does not have any form of the finite model property for these two semantics, we are able to prove decidability of the validity problem. The proof makes use of *quasimodels* [1], which are a variation on Kripke models where time can be nondeterministic. We can show that every falsifiable formula is falsifiable on a finite quasimodel, which yields decidability. We then strengthen this result to PSPACE-complete. Further, we provide a deductive calculus for Gödel temporal logic with a finite number of axioms and deduction rules, and can show this calculus to be sound and complete for the abovementioned semantics.

[1] David Fernández-Duque, *Non-deterministic semantics for dynamic topological logic*, ***Annals of Pure and Applied Logic***, vol. 157 (2009), no. 2–3, pp. 110–121.

▸ DJAMEL EDDINE AMIR*, AND MATHIEU HOYRUP, *Computability of finite simpilicial complexes and homology.*
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The topological properties of a space have a strong impact on its computability properties. The notion of computable type is an interesting example of this fact. A space has computable type if every effectively compact copy of it is computable. Many spaces have this property, such as spheres and closed manifolds [1, 2]. A similar notion is defined for pairs with computable type.

We proved recently a characterization of simplicial pairs with computable type (see [3]). In particular, we proved that a simplicial cone pair has computable type iff it has the surjection property. Namely, a simplicial pair $(Cone(X), X)$ has computable type iff every continuous function $f : Cone(X) \to Cone(X)$ which is the identity in X is a surjection.

It raises a purely topological question: when does a pair has the surjection property? We prove connections between the surjection property and homology: for instance, the cone of a graph has the surjection property iff every point of its base is in a cycle. We try to generalize this to some other simplicial pairs and we explain an open question whose positive answer gives a full characterization of simpilicial cone pairs which have the surjection property using the relative homology of their base pairs.

[1] Joseph S. Miller, *Effectiveness for Embedded Spheres and Balls*, ***Electronic Notes in Theoretical Computer Science***, vol. 66 (2002), pp. 127–138.

[2] Zvonko Iljazović and Igor Sušić, *Semicomputable manifolds in computable topological spaces*, ***Journal of Complexity***, vol. 45 (2018), pp. 83–114.

[3] Djamel Eddine Amir and Mathieu Hoyrup, *Computability of finite simplicial complexes*, (2022), arxiv:2202.04945.

▸ JONAS RAFAEL BECKER ARENHART AND HITOSHI OMORI*, *More on Constructive Nonsense Logic.*
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In [4], Peter Woodruff devised a constructive version of Sören Halldén's logic of nonsense, presented in [2], with an additional connective introduced by Krister Segerberg in [3]. Woodruff's project is extremely rich and fruitful, but when seen in light of the original motivations set forth by Halldén, the suggested semantics seems to be not without problems. More specifically, a sentence is understood as meaningless in the original reading provided by Halldén when it is neither true nor false, but Woodruff's semantics will allow what should count as prima facie meaningless sentences to receive one of the truth values. The problem is that in the original reading by Halldén 'meaningless' is defined as 'non truth evaluable' so that a kind of tension between the original motivation and the constructive approach by Woodruff appears explicitly.

Then, it seems that we will face at least the following two questions.

Q1: How can we formulate a constructive version of Halldén's logic that reflects Halldén's understanding of meaninglessness?

Q2: How can we make sense of formal systems along suggestions made by Woodruff, if the meaningless reading is not a good fit?

The aim of this paper is to address the second question, and to this end, we will build on a reading of Weak Kleene Logic suggested by Jc Beall in [1]. Instead of using the meaningful/meaningless dichotomy, we follow Beall and introduce the on-topic/off-topic distinction, so that a sentence may be, in the end, true or false, and also on-topic or off-topic. That kind of move avoids the trouble caused by Woodruff's original reading, with advantages that shall also be explored in this paper.

[1] Jc Beall, *Off-topic: A new interpretation of weak-Kleene logic*, ***The Australasian Journal of Logic***, vol. 13 (2016), no. 6, pp. 136–142.

[2] Sören Halldén, ***The Logic of Nonsense***, Uppsala Universitets Årsskrift, 1949.

[3] Krister Segerberg, *A contribution to nonsense-logics*, ***Theoria***, vol. 31 (1965), no. 3, pp. 199–217.

[4] Peter Woodruff, *On constructive nonsense logic*, ***Modality, morality, and other problems of sense and nonsense: Essays dedicated to Sören Halldén*** GWK Gleerup Bokforlag, Lund, 1973, pp. 192–205.

► GUILLERMO BADIA*, XAVIER CAICEDO, AND CARLES NOGUERA, *Frame definability in finitely-valued modal logic*.

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In this paper we study frame definability in finitely-valued modal logics and establish two main results via suitable translations: (1) in finitely-valued modal logics one cannot define more classes of frames than are already definable in classical modal logic (cf. [2, Thm. 8]), and (2) a large family of finitely-valued modal logics define exactly the same classes of frames as classical modal logic (including modal logics based on finite Heyting and MV-algebras). In this way one may observe, for example, that the celebrated Goldblatt–Thomason theorem applies immediately to these logics. In particular, we obtain the central result from [1] with a much simpler proof and answer one of the open questions left in that paper. Moreover, the proposed translations allow us to determine the computational complexity of a big class of finitely-valued modal logics. Finally, we show that the first translation we offer (from finitely-valued modal logic into two-valued modal logic) yields a 0-1 law over models for the former (cf. [3]) as a corollary of W. Oberschelp's generalization [4] of Fagin's 0-1 law. In particular, one can show that, over Kripke models for finitely-valued modal logics based on finite frames, for every modal formula there is a truth-value that it takes almost surely at all worlds.

[1] B. Teheux. Modal definability for Łukasiewicz validity relations. *Studia Logica* 104 (2): 343–363 (2016).

[2] S.K. Thomason. Possible worlds and many truth values. *Studia Logica* 37: 195–204 (1978).

[3] J. Y. Halpern and B. M. Kapron, Zero-one laws for modal logic, *Annals of Pure*

and Applied Logic, 69: 157–193 (1994).

[4] Walter Oberschelp. Asymptotic 0-1 laws in combinatorics. In D. Jungnickel (ed.), *Combinatorial theory, Lecture Notes in Mathematics* 969:276–292, Springer, 1982.

▸ BRUNO BENTZEN, *Can propositions be intentions, intuitionistically?*
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Dissatisfaction with the philosophical thought of L. E. J. Brouwer has led to a growing interest over the last few decades in the support of his intuitionism from a phenomenological approach, building on ideas from Husserl. The main supporters of this interpretation are Richard Tieszen and Mark van Atten. It is rooted in Heyting's idea that a proposition is an intention which is fulfilled with a proof-object of it.

In this talk I argue against this propositions-as-intentions interpretation. I must stress at the outset that the interpretation is already a target of harsh criticisms regarding the incompatibility of Brouwer's and Husserl's positions, mainly from Guillermo Rosado Haddock or Claire Hill. But their objection consists in denying the interpretation its major premise.

This is not the direction I wish to take in this talk. Instead, I object that even if we grant that the incompatibility can be properly dealt with, as van Atten believes it can, one fundamental issues remain: it is far from clear what the object of an intention corresponding to a proposition should be. I argue that Heyting's own suggestion is inadequate and the most plausible candidates for intentional objects are sets of canonical proof-objects of the propositions. But this thesis immediately leads us to a difficult fulfillment dilemma: for Husserl, an intention is fulfilled when the intended object is genuinely presented to us in just the way it is intended; but here only one element of the set, not the set itself, can fulfill the intention. I conclude that the propositions-as-intentions leads to undesirable consequences.

▸ KATALIN BIMBÓ, *Relational semantics for some classical relevance logics.*
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The framework called *generalized Galois logics* (or gaggle theory, for short) was introduced in [2] to encompass Kripke's semantics for modal and intuitionistic logics, Jónsson & Tarski's representation of BAO's and the Meyer–Routley semantics for relevance logics among others. In some cases, gaggle theory gives exactly the semantics defined earlier for a logic; in other cases, the semantics differ (cf. [3], [1]). Relational semantics for classical relevance logics such as **CR** and **CB** are usually defined as a modification of the Meyer–Routley semantics for $\mathbf{R}_+$ and $\mathbf{B}_+$, respectively (cf. [4]). In this talk, I compare the existing semantics for **CB** and **CR** to the semantics that results as an application of gaggle theory.

[1] Bimbó, Katalin and J. Michael Dunn, ***Generalized Galois Logics: Relational Semantics of Nonclassical Logical Calculi***, CSLI Lecture Notes vol. 188, CSLI Publications, Stanford, CA, 2008.

[2] Dunn, J. Michael, *Gaggle theory: An abstraction of Galois connections and residuation, with applications to negation, implication, and various logical operators*, ***Logics in AI: European Workshop JELIA '90***, (J. van Eijck, editor), Lecture Notes in Computer Science vol. 478, Springer, Berlin, 1991, pp. 31–51.

[3] Dunn, J. Michael, *Gaggle theory applied to intuitionistic, modal and relevance logics*, ***Logik und Mathematik. Frege-Kolloquium Jena 1993***, (I. Max and W. Stelzner, editors), W. de Gruyter, Berlin, 1995, pp. 335–368.

[4] Meyer, Robert K., *Ternary relations and relevant semantics*, ***Annals of Pure and Applied Logic***, vol. 127 (2003), pp. 195–217.

▶ JASON BLOCK, *Categoricity ordinals and models of Presburger arithmetic.*
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The categoricity ordinal of a structure $\mathcal{M}$ is a measure of how hard it is to compute isomorphisms between copies of $\mathcal{M}$. Presburger arithmetic is the theory of $(\mathbb{Z}, +, <)$. We will precisely define categoricity ordinals, explore computability-theoretic properties of Presburger arithmetic, and examine which ordinals can be the categoricity ordinal for a model of Presburger arithmetic.

▶ MARIA BEATRICE BUONAGUIDI, *Symmetry, locality and hyperintensionality.*
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The notion of hyperintensionality and has become of prime importance in contemporary research, to the point that Nolan [3, p. 149] predicted a "hyperintensional revolution" for the 21st century. In a recent paper, Odintsov and Wansing [4] criticize the claim to hyperintensionality of Leitgeb's logic HYPE [2], and suggest a definition of hyperintensionality based on a logic's consequence relation [4, p. 51]. According to this criterion, HYPE is not hyperintensional, but merely intensional. However, the picture is not as clear-cut as it seems. Indeed, we can distinguish three notions of hyperintensionality, corresponding to different informal definitions in the hyperintensional metaphysics literature: Odintsov and Wansing only formulate one of these criteria. However, HYPE can be assessed also with respect to the other two. I show it to be hyperintensional with respect to the last, and weakest, criterion. We cannot therefore say that HYPE is hyperintensional proper, but we can say that it is strongly intensional, and not just intensional. In fact, it is precisely the fact that HYPE is strongly intensional and not merely intensional allows to successfully model hyperintensional operators in HYPE-models. This shows that, although HYPE's consequence relation is well-behaved enough to make it stronger than most non-classical logics, HYPE's semantics is especially powerful.

I note another result showing this feature. Leitgeb argued that HYPE has the disjunction property [2, p. 346]. However, as noted by Odintsov and Wansing [4, p. 43], HYPE can be shown to be equivalent to the logic N_i^*, which has been shown not to have the disjunction property [1, p. 400]. I show that HYPE does not have the disjunction property at the level of its consequence relation, but that the disjunction property holds in all HYPE-models.

[1] Sergey A. Drobyshevich, *Double negation operator in logic N^**, ***Journal of mathematical sciences***, vol. 205 (2015), no. 3, pp. 389–402.

[2] Hannes Leitgeb, *HYPE: a system of hyperintensional logic*, ***Journal of philosophical logic***, vol. 48 (2019), no. 2, pp. 305–405.

[3] Daniel Nolan, *Hyperintensional metaphysics*, ***Philosophical Studies***, vol. 171 (2014), no. 1, pp. 149–160.

[4] Sergey Odintsov and Heinrich Wansing, *Routley star and hyperintensionality*, ***Journal of philosophical logic***, vol. 50 (2021), no. 1, pp. 39–56.

▶ YONG CHENG, *The relevance of the incompleteness theorems with Hilbert's concrete proof theory.*
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It is widely believed that Gödel's first and second incompleteness theorem (G1 and G2) undermined Hilbert's program. We examine the relevance of G1 and G2 with Hilbert's concrete proof theory. We argue that even if G1 and G2 refute (in a narrow sense) some original goals of Hilbert's program, Gödel's solutions are not concrete and real in the sense of Hilbert's concrete proof theory.

We could view G1 as an existence problem. In the sense of Hilbert's concrete proof theory, the independent sentence of PA Gödel constructed is an ideal element which is not real and concrete. Even if we can say (in the narrow sense) that G1 shows there is no strong enough consistent axiomatized formal system in which all true statements are provable, G1 does not answer the following question in the spirit of Hilbert's concrete proof theory: whether all concrete true arithmetic sentences are provable in PA. The research program after Gödel on concrete incompleteness looks for concrete and real solutions of the existence problem of G1. We could view this research practice as a realization of Hilbert's concrete proof theory.

It is a popular view that G2 destroys Hilbert's consistency program. Nonetheless, there are dissidents (see [2], [1]). Neither Gödel nor Hilbert think Hilbert's consistency program were destroyed by G2. Hilbert thinks G2 only shows one must exploit the finitary standpoint in a sharper way for the consistency proofs (see [4]). Gödel writes in [3] that it is conceivable that there exist finitary proofs that can not be expressed in the formalism of the basis system. We argue that the current research on the consistency problem confirms Gödel's view that G2 does not destroy but leaves Hilbert's program very much alive and even more interesting than it initially was. There is no purely mathematical solution of the consistency problem since each solution of it is related to a philosophical question: what is a solution of the consistency problem? A key issue of this problem is: how to formulate the consistency statement and what is the "correct" formulation if any? Different formulations of the consistency statement may lead to different answers of the consistency problem. We examine different methods to formulate the consistency statement and compare them in the spirit of Hilbert's concrete proof theory: whether one formulation of the consistency statement is more concrete than another one. This research is a beginning step toward the interesting open question: what is a "natural" consistency statement?

In summary, in a strong sense we argue that Gödel's original G1 and G2 have no relevance with Hilbert's concrete proof theory; but some lines of research after Gödel on G1 and G2 can be interpreted as a realization of Hilbert's concrete proof theory in the sense of finding concrete solutions of the existence problem in G1 and formulating the consistency statement in a more natural and concrete way.

[1] Sergei Artemov. The Provability of Consistency. preprint, see arXiv: 1902.07404v5, 2019.

[2] M. Detlefsen. What does Gödel's second theorem say? Philosophia Mathematica, 9:37-71, 2001.

[3] K. Gödel. On formally undecidable propositions of Principia Mathematica and related systems. In Collected Works: Oxford University Press: New York. Editor-inchief: Solomon Feferman. Volume I: Publications 1929-1936, 1986.

[4] D. Hilbert and P. Bernays. Grundlagen der Mathematik. Vol. I. Springer, 1934.

► SZYMON CHLEBOWSKI AND PATRYCJA KUPŚ*, *Meaning is Use: the Case of Propositional Identity.*
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We study natural deduction system for a fragment of intuitionistic logic with propositional identity from the point of view of proof-theoretic semantics. In this logic the propositional identity connective is established only by elimination rules, that is it cannot be asserted under any conditions, thus it is incompatible with the Gentzen approach. Following Schroeder-Heister, we define two types of validity: introduction- and elimination-rule based. We argue that the identity connective is a natural operator to be treated by the elimination rules as basic approach. Moreover, we show that it does not change even if the introduction rule for the identity connective is formulated.

[1] Bloom, S. L. and Suszko, R., *Investigations into the Sentential Calculus with Identity*, ***Notre Dame Journal of Formal Logic***, vol. 13 (1972), no. 3, pp. 289–308.

[2] Gentzen, G., *Investigations into logical deductions*, ***The collected papers of Gerhard Gentzen*** (M. E. Szabo, editor), Elsevier Science, 1969, pp. 68–131.

[3] Negri, S. and von Plato, J., ***Proof analysis, a contribution to hilbert's last problem***, Cambridge University Press, 2011.

[4] Schroeder-Heister, P., *Validity concepts in proof-theoretic semantics*, ***Synthese***, vol. 148 (2006), pp. 525 –571.

[5] ——— *Proof Theoretical Validity Based on Elimination Rules*, ***Why is this a Proof? Festschrift for Luiz Carlos Pereira*** (E.H. Haeusler, W. de Campos Sanz and B. Lopez, editors), College Publications, 2015, pp. 159–176.

▸ SZYMON CHLEBOWSKI*, MARTA GAWEK, AGATA TOMCZYK, *Propositional identity. From semantics to proof theory.*
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The aim of the talk is to discuss differences between formal and philosophical interpretations of propositional identity across classical and intuitionistic logic. In classical logic propositional identity is closely related to the abolition of the Fregean Axiom [2], according to which sentences are names of truth values. In reference to the ideas from Wittgenstein's *Tractatus* we may, contrary to Frege, assume that propositions denote situations. In such case, due to Quine *dictum*, we need to introduce criteria of identity of situations, as it was done for example by Suszko [3]. This picture changes when we move to the constructive environment. Here, propositions can be thought of as types of their own proofs and propositional identity can be interpreted as expressing the notion of identity of proofs [1].

[1] Szymon Chlebowski and Dorota Leszczyńska-Jasion, *An Investigation into Intuitionistic Logic with Identity*, ***Bulletin of the Section of Logic***, vol. 48 (2019), no. 4, pp. 259–283.

[2] Roman Suszko, *Abolition of the Fregean Axiom*, ***Lecture Notes in Mathematics***, vol. 453 (1975), pp. 169–239.

[3] Roman Suszko, *Ontology in the Tractatus of L. Wittgenstein*, ***Notre Dame Journal of Formal Logic***, vol. 9 (1968), no. 1 pp. 7–33.

▸ CEZARY CIEŚLIŃSKI, *Two halves of disjunctive correctness.*
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The disjunctive correctness principle (DC) states that a disjunction of arbitrary length is true if and only if one of its disjuncts is true. On first sight, the principle seems an innocent and natural generalization of the familiar compositional truth axiom for disjunction, which states that a disjunction of *two* sentences is true if and only if one of them is true. Since the generalized version applies to disjunctions of arbitrary lengths, it can be applied also in non-standard models of arithmetic, where some disjunctions will have a non-standard length.

Ali Enayat and Fedor Pakhomov (see [1]) demonstrated that adding (DC) to the classical compositional truth theory CT^- permits to prove Δ_0 induction for the language with the truth predicate, hence it produces a non-conservative extension of the background arithmetical theory (see [2]).

We will present the proof of a stronger result. Let (DC-Elim) be just one direction of (DC), namely, the implication "if a disjunction is true, then one of it disjuncts is true". We will show that already (DC-Elim) carries the full strength of Δ_0 induction; moreover, the proof of this fact will be significantly simpler than the original argument of Enayat and Pakhomov.

Let (DC-intro) be the opposite direction of (DC), namely, the implication "if a given sentence φ is true, then a disjunction having φ as a disjunct is true". Unlike (DC-Elim), (DC-intro) can be conservatively added to the truth axioms of CT^-.

[1] Enayat, Ali and Pakhomov, Fedor, *Truth, disjunction, and induction*, ***Archive for Mathematical Logic***, vol. 58 (2019), pp. 753–766.

[2] Łełyk, Mateusz and Wcisło, Bartosz, *Notes on bounded induction for the composi- tional truth predicate*, ***Review of Symbolic Logic***, vol. 10 (2017), pp. 355–480.

► PETR CINTULA, *Abstract Lindenbaum lemma for non-finitary consequence relations.*
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The Lindenbaum lemma is an easy yet crucial result in algebraic logic abstractly formulated as: for any *finitary* consequence relation, the meet-irreducible theories form a basis of the closure system of its theories. While the finitarity restriction is crucial for its usual proof, it is not necessary: there are works (e.g. [3, 4, 5]) proving it (or its variant for *finitely* meet-irreducible theories) for certain *infinitary structural* consequence relations. The paper [1] provides a general result (covering most of the known cases) for *structural* consequence relations with a *countable* Hilbert-style axiomatization and a *strong disjunction* [2]. Identifying the essential non-structural properties of strong disjunctions we can prove:

Lemma 1. *Let $\vdash$ be a consequence relation with a countable axiomatization such that the closure system $\mathcal{T}_\vdash$ of its theories is a frame (i.e. satisfies the corresponding infinite distributive law) and the intersection of two finitely generated theories is finitely generated. Then the finitely meet-irreducible theories form a basis of $\mathcal{T}_\vdash$.*

[1] Marta Bílková, Petr Cintula, Tomáš Lávička, *Lindenbaum and pair extension lemma in infinitary logics*, ***WoLLIC 2018***, (Moss, de Queiroz, Martinez, editors), Springer, 2018, pp. 134–144.

[2] Petr Cintula, Carles Noguera, ***Logic and Implication: An Introduction to the General Algebraic Study of Non-classical Logics***, Springer, 2021.

[3] Robert Goldblatt, ***Mathematics of Modality***, CSLI Publications Stanford University, 1993.

[4] Krister Segerberg, *A model existence theorem in infinitary propositional modal logic*, ***Journal of Philosophical Logic***, vol. 23 (1994), pp. 337–367.

[5] Göran Sundholm, *A completeness proof for an infinitary tense-logic*, ***Theoria***, vol. 43 (1977), pp. 47–51.

► VITTORIO CIPRIANI, *Equivalence relations and learning of algebraic structures.*
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In this talk we present some results related to algorithmic learning of algebraic structures. In a series of papers [FKSM19, BFSM20, BSM21] the authors developed a framework in which a learner receives larger and larger pieces of an arbitrary copy of a computable structure and, at each stage, is required to output a conjecture about the isomorphism type of such a structure. The learning is successful if the conjectures eventually stabilize to a correct guess. Borrowing ideas from descriptive set theory, we aim to calibrate the complexity of nonlearnable families, offering a new hierarchy based on reducibility between equivalence relations. To do so, we define the notion of E-learnability.

Definition 1. A family of structures $\mathfrak{K}$ is E-learnable if there is function $\Gamma : 2^\omega \to 2^\omega$ which continuously reduce $\mathsf{LD}(\mathfrak{K})_{/\cong}$ to E, where $\mathsf{LD}(\mathfrak{K})$ is the collection of all copies of the structures from $\mathfrak{K}$.

For example, we show that the paradigm introduced at the beginning coincides with E_0-learnability, where E_0 is the eventual agreement on reals. We then focus on the learning power of well-known benchmark Borel equivalence relations differentiating between learnability of finite and countably infinite families. The work presented in this talk is a joint work with Nikolay Bazhenov and Luca San Mauro, and some of the results discussed here can be found in [BCSM21].

[BFSM20]Nikolay Bazhenov, Ekaterina Fokina, and Luca San Mauro. Learning families of algebraic structures from informant. *Information and Computation*, 275:104590, 2020.

[FKSM19]Ekaterina Fokina, Timo Kötzing, and Luca San Mauro. Limit learning equivalence structures. In Aurélien Garivier and Satyen Kale, editors, *Proceedings of the 30th International Conference on Algorithmic Learning Theory*, volume 98 of *Proceedings of Machine Learning Research*, pages 383–403, Chicago, Illinois, 22–24 Mar 2019. PMLR.

[BSM21]Nikolay Bazhenov and Luca San Mauro. On the Turing complexity of learning finite families of algebraic structures. *Journal of Logic and Computation*, 2021. Published online. arXiv preprint arXiv:2106.14515.

[BCSM21]Nikolay Bazhenov, Vittorio Ciprani and Luca San Mauro. Learning algebraic structures with the help of Borel equivalence relations. arXiv preprint available at arxiv.2110.14512.

▸ LUDOVICA CONTI, *Arbitrary Abstraction and Logicality.*
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In this talk, I will discuss a criterion (weak invariance) that has been recently suggested in order to argue for the logicality of abstraction operators, when they are understood as arbitrary expressions (cf. Boccuni Woods 2020). The issue of logicality of the abstractionist vocabulary was originally raised within the seminal abstractionist program, Frege's Logicism, and represents, still today, a crucial topic in the abstractionist debate. My double aim consists in inquiring this topic both from a formal and from a philosophical point of view.

On the one side, I will argue that, while weak invariance is not satisfied (except for specific exceptions, cf. [4], [3]) by first-order abstraction principles (APs), it characterises a wide range of higher-order ones. More precisely, by comparing respective schemas of first-order and second-order APs, we will note that logicality (in the chosen meaning) mirrors a relevant distinction between same-order and different-order abstraction principles. So, after discussing the controversial case of Ordinal Abstraction, I will note that, if we accept an arbitrary interpretation of APs, not only Neologicism (based on HP), but many current abstractionist programs and even the consistent revisions of Frege's Logicism (based on weakened versions of BLV) are able to achieve the logicality objective.

On the other side, from a philosophical point of view, I will discuss the role of arbitrariness as a condition for the adoption of the abovementioned logicality criterion. Particularly, I will argue that, on the one hand, the arbitrary interpretation could be considered as the most faithful to abstractionist theories, but, on the other hand, it includes semantic insights that are radically alternative to Logicism. In order to argue for this latter consideration, an analogy between the arbitrary interpretation of the APs and the semantics of some eliminative structuralist reconstructions of the scientific theories will be illustrated.

[1] Francesca Boccuni, Jack Woods, *Structuralist Neologicism* ***Philosophia Mathematica***, (2018), 28(3), 296-316.

[2] Kit Fine, *The limits of Abstraction* **Clarendon Press**, (2002).

[3] Jack Woods, *Logical indefinites , 277-307* ***Logique et Analyse***, (2014) pp. 277-307.

[4] Gabriel Uzquiano *The concept of truth in formalized languages* ***Logic, semantics, metamathematics***, (1956) 2(152-278), 7.

▸ TONICHA CROOK*, JAY MORGAN, MARKUS ROGGENBACH AND ARNO PAULY,
A computability perspective on verified machine learning.
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We approach the idea of verified machine learning from the perspective of computable analysis. By using the language of computable analysis, particularly that of represented spaces, we can formalize various verification questions of relevance for the machine learning community. These formulations involve function spaces, as well as the spaces of open subsets, overt subsets and compact subsets. We can then show that as long as the appropriate notions of subsets are chosen, and as long as we use ternary logic including an undetermined truth value, these verification questions do become computable.

[1] Tonicha Crook, Jay Morgan, Markus Roggenbach and Arno Pauly, *Computability Perspective on (Verified) Machine Learning*, ***arXiv 2102.06585***,

▸ ANUPAMN DAS AND AVGERINOS DELKOS*, *Proof complexity of monotone branching programs.*
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We investigate the proof complexity of systems based on positive branching programs, i.e. non-deterministic branching programs (NBPs) where, for any 0-transition between two nodes, there is also a 1-transition. Positive NBPs compute monotone Boolean functions, like negation-free circuits or formulas, but constitute a positive version of (non-uniform) $\mathbf{NL}$, rather than $\mathbf{P}$ or $\mathbf{NC}^1$, respectively.

The proof complexity of NBPs was investigated in previous work by Buss, Das and Knop, using extension variables to represent the dag-structure, over a language of (non-deterministic) decision trees, yielding the system eLNDT. Our system eLNDT^+ is obtained by restricting their systems to a positive syntax, similarly to how the 'monotone sequent calculus' MLK is obtained from the usual sequent calculus LK by restricting to negation-free formulas.

Our main result is that eLNDT^+ polynomially simulates eLNDT over positive sequents. Our proof method is inspired by a similar result for MLK by Atserias, Galesi and Pudlák, that was recently improved to a bona fide polynomial simulation via works of Jeřábek and Buss, Kabanets, Kolokolova and Koucký. Along the way we formalise several properties of counting functions within eLNDT^+ by polynomial-size proofs and, as a case study, give explicit polynomial-size poofs of the propositional pigeonhole principle.

▸ PABLO DOPICO, *Truth-theoretic determinacy revisited.*
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Despite being highly successful, Saul Kripke's theory of truth (1975), based on the so-called fixed-point semantics, has been criticised on the basis of its incapacity to formulate the semantic status of paradoxical sentences such as the Liar. In other words, Kripke's theory treats that and similar sentences as being neither true nor false, but the object language lacks the resources to speak about the *gappy* character of the Liar. From Burge (1979) to Field (2008), the tradition has suggested that this gappy character can be best understood as the idea that the Liar is not determinate or not determinately true. As a result, the main aim of this paper is to explore what being determinate in relation to a truth predicate could mean. We hence propose three different understandings of such notion in the form of three determinacy predicates and offer philosophical motivations for each of them. After that, we test different theories of truth against the background of those three understanding of truth-theoretic determinacy. In particular, we assess Kripke's theory of truth, as well as Solomon Feferman's well-known axiomatization of it (the so-called **KF**) (Halbach 2014), and Vann McGee's theory of definite truth (1991). Our results suggest that there could be a trade-off between the semantic expressibility of a theory of truth, understood as its ability to capture the semantic status of the Liar, and the logical strength of the theory.

[1] Saul Kripke, *Outline of a theory of truth*, ***The Journal of Philosophy***, vol.72, no.19, pp.690-716.

[2] Tyler Burge, *Semantical paradox*, ***The Journal of Philosophy***, vol.76, no.4, pp.169-198.

[3] Hartry Field, ***Saving truth from paradox***, Oxford University Press, 2008.

[4] Volker Halbach, ***Axiomatic theories of truth***, Cambridge University Press, 2014.

[5] Vann McGee, ***Truth, vagueness, and paradox: An essay on the logic of truth***, Hackett Publishing, 1991.

▸ HSING-CHIEN TSAI, ZE-YUAN DUAN, *1-Complete first-order mereological theories.*
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Previously we have shown that a lot of first-order axiomatizable mereological theories are undecidable [1]. Since any first-order axiomatizable theory is recursively enumerable and it is known in the theory of computability that K is the hardest recursively enumerable set in terms of Turing reduction, it is immediately an interesting question whether each of those undecidable first-order axiomatizable mereological theories is as hard as K, or in other words, is Turing equivalent to K. Since any 1-complete set must be Turing equivalent to K, we will give a positive answer to the said question by showing that each of those undecidable first-order axiomatizable mereological theories is 1-complete.

[1] Hsing-Chien Tsai, *A Comprehensive Picture of the Decidability of Mereological Theories*, ***Studia Logica***, vol. 101 (2013), no. 5, pp. 987-1012.

► FREDRIK ENGSTRÖM, *Foundations of team semantics.*
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Dependence logic, see [3], and its relatives are defined using team semantics, in which formulas are satisfied by sets of assignments, teams, rather than single assignments. The team semantics construction is widely applicable and can be used to understand notions from many different areas; model theory, game theory, database theory, probabilistic reasoning and program verification, to name a few.

The denotatiton of a first-order formula in classical Tarskian semantics is the set of assignments satisfying the formula, $[\![\varphi]\!]_c$. In team semantics it is the set of teams satisfying the formula, i.e., a set of sets of assignments, $[\![\varphi]\!]_t$. The standard team semantic construction is via the *flatness principle* according to which $[\![\varphi]\!]_t = \mathcal{P}([\![\varphi]\!]_c)$.

This construction can, at least partially, be described using the free functor from the category of partially ordered monoids to the category of quantales, i.e., partially ordered monoids equipped with a complete semilattice structure, see [1]. This functor maps the space of Tarskian denotations, $\mathcal{P}(X^V)$, where X is the domain and V a set of variables, into the space $\mathcal{H}(\mathcal{P}(X^V))$, the set of downwards-closed subsets of $\mathcal{P}(X^V)$. The embedding is based on the flatness principle in that $[\![\varphi]\!]_t = \mathcal{P}([\![\varphi]\!]_c)$.

However, the space of downwards-closed sets can not be used as the space of denotations for some logics: One example is the well-studied Independence logic, which isn't downward-closed; another is a logic constructed to handle branching of non-monotone generalized quantifiers, which isn't based on the flatness principle. I will in this talk revisit the description of the team semantic construction as the free functor from a more general perspective that also includes these logics.

[1] Samson Abramsky and Jouko Väänänen, *From IF to BI*, ***Synthese***, vol. 167 (2009), pp. 207–230.

[2] Fredrik Engström, *Generalized quantifiers in Dependence logic*, ***Journal of Logic, Language and Information***, vol. 21 (2012), pp. 299–324.

[3] Jouko Väänänen, ***Dependence logic. A new approach to independence friendly logic***, London Mathematical Society Student Texts, Cambridge University Press, 2007.

► DAVID FERNÁNDEZ-DUQUE, ORIOLA GJETAJ*, ANDREAS WEIERMANN, *Intermediate Goodstein principles.*
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The Goodstein principle is a natural number-theoretic theorem which is unprovable in Peano arithmetic.The original process proceeds by writing natural numbers in nested exponential k-base normal form, then successively raising the base to k + 1 and subtracting one from the end result. Such sequences always reach zero, but this fact is unprovable in Peano arithmetic.

In this talk, we will consider canonical representations of natural numbers using Ackermann function and the function of Grzegorczyk hierarchy. These representations give a natural Goodstein process for which we obtain independence from different theories of reverse mathematics.

This is a joint ongoing work with A. Weiermann and D. Fernández-Duque on exploring normal form notations for the Goodstein principle.

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▶ DAVID FERNÁNDEZ-DUQUE, JOOST J. JOOSTEN, AND KONSTANTINOS PAPAFILIPPOU*,
Hyperarithmetical Worm Battles.
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Japaridze's provability logic GLP has one modality $[n]$ for each natural number and has been used by Beklemishev for a proof theoretic analysis of Peano aritmetic (PA) and related theories. In his analysis, he interprets GLP in arithmetic by interpreting each modality $\langle n \rangle$ as Σ_n-RFN$(T) := \{\Box_T \phi \to \phi : \phi$ is a Σ_n formula$\}$, for some given theory T and with $\Box_T \phi$ standing for the formula: "ϕ is provable in T". He examines what he calls worms, which is the set W of formulas in GLP defined as:

- $\top \in \mathsf{W}$;
- if $A \in \mathsf{W}$ and n is a natural number, then $\langle n \rangle A \in \mathsf{W}$.

Among other benefits, this analysis yields [1] the so-called *Every Worm Dies* (EWD) principle, a natural combinatorial statement that is similar in spirit to the hercules hydra battle and a bit more closely connected to the assertion of the totality of Hardy functions H_α on ordinals $\alpha < \epsilon_0$. He had then proven that EWD is equivalent over EA $:= \mathsf{I\Delta_0} + \mathsf{exp}$ to the Σ_1-RFN(PA) and hence it is also independent of PA. Recently, Beklemishev and Pakhomov [2] have studied notions of provability corresponding to transfinite modalities in GLP and they have looked into their connection to some theories of second order arithmetic. We show [3] that indeed the natural transfinite extension of GLP is sound for this interpretation, and yields similarly an equivalence to the Σ_1-reflection of the second order theory ACA of arithmetical comprehension with full induction. We also provide restricted versions of EWD related to the fragments $I\Sigma_n$ of Peano arithmetic.

[1] L D Beklemishev, *Reflection principles and provability algebras in formal arithmetic*, ***Russian Mathematical Surveys***, vol. 60 (2005), no. 2, pp. 197–268.

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[3] Fernández-Duque, David and Papafilippou, Konstantinos and Joosten, Joost J., *Hyperarithmetical Worm Battles*, ***Logical Foundations of Computer Science*** (Cham), (Artemov Sergei and Nerode Anil, editors), vol. 13137, Publisher Springer International Publishing, Year 2022, pp. 52–69.

▸ CHRISTOF FETZER AND MUHAMMAD USAMA SARDAR*, *Understanding trust assumptions for remote attestation via formal verification.*
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Trust is a very critical and yet one of the least understood processes in the computing paradigm. As opposed to typical case studies based on toy examples, we demonstrate how we leverage formal verification to understand the complicated notion of trust in the real-world settings, with a specific focus on remote attestation in confidential computing. In this talk, we present the challenges and lessons learnt in the formal specification and verification of Intel's next generation architecture named Intel Trust Domain Extensions, and demonstrate how we ended up making Intel update the specification.

The proposed talk will specifically address the following questions:

- What is confidential computing? How does it compare with the existing state-of-the-art technologies, such as Homomorphic Encryption?
- Why remote attestation is critical in confidential computing?
- What were the challenges in the formal specification of remote attestation in Intel Software Guard Extensions (SGX) and the upcoming Intel Trust Domain Extensions (TDX)?
- How we drive formal methods to practice for the automated verification of security properties of remote attestation protocols in Intel SGX and TDX?
- What are interesting open challenges of relevance for logic community for formal verification of remote attestation in confidential computing?

▸ FRANCESCO GALLINARO, *Exponential sums equations and tropical geometry.*
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In the late 1990s, Boris Zilber made a conjecture on the model theory of the exponential function, the Quasiminimality Conjecture (see [2], [3]). This predicts that all subsets of the complex numbers that are definable using the language of rings and the exponential function are either countable or cocountable. He then proved that the conjecture would follow if the complex exponential field were a model of a certain theory in an infinitary logic. Building on Zilber's work, Bays and Kirby have proved in [1] that the Quasiminimality Conjecture would follow from just one of Zilber's axioms, the Exponential-Algebraic Closedness Conjecture, which predicts sufficient conditions for systems of equations in polynomials and exponentials to have complex solutions. In this talk, I will give an introduction to this topic before presenting some recent work which solves the conjecture for a class of algebraic varieties which corresponds to systems of exponential sums. This turns out to be closely related to tropical geometry, a "combinatorial shadow" of algebraic geometry which reduces some questions about algebraic varieties to questions about polyhedral objects.

[1] Martin Bays and Jonathan Kirby, *Pseudo-exponential maps, variants, and quasiminimality*, ***Algebra & Number Theory***, vol.12 (2018), no.3, pp.493-549.

[2] Boris Zilber, *Analytic and pseudo-analytic structure*, ***Lecture Notes in Logic, 19. Logic Colloquium 2000, Paris***, (Rene Cori, Alexander Razborov, Stevo Todorčević, and Carol Wood, editors), Cambridge University Press, 2005, pp.392-408.

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▸ MICHAŁ TOMASZ GODZISZEWSKI, *Between the Model-Theoretic and the Axiomatic Method of Characterizing Mathematical Truth.*
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The so-called model-theoretic method of characterizing the notion of truth consists in defining a general notion of a model of a given formal language L, providing a definition of a binary relation between models of L and the sentences of L, and finally singling out a concrete model as the standard or the intended one and declaring that truth simpliciter (of sentences of L) should be understood as truth in this model. Can we really treat this model-theoretic definition of truth as the definition of (mathematical) truth (say, at least with respect to the language of arithmetic)? There are at least two serious problems with this method, one of which is that even if we might assume that our metatheory can provide us with a determinate concept of the standard model, then the question is: does it follow that then the concept of truth is definite, complete, determinate or absolute? In what follows, we provide an analysis of these two particular questions regarding the use of the notion of standard model in the model-theoretic characterization of the notion of mathematical truth simpliciter, leading to results that can be interpreted as delivering the following message: not only there are conceptual problems regarding the way standard models are used in the characterization, but there are philosophically justified mathematical reasons for which the appeal to standard models in truth- theoretic constructions is at least problematic, if not impossible, and therefore, if one's goal is to provide a formal theory of mathematical truth simpliciter, the axiomatic framework is the right method of doing so.

▸ MICHAŁ TOMASZ GODZISZEWSKI, *Tennebaum's Theorem for quotient presentations of nonstandard models of arithmetic.*
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A computable quotient presentation of a mathematical structure $\mathcal{A}$ consists of a computable structure on the natural numbers $\langle \mathbb{N}, \star, *, \dots \rangle$, meaning that the operations and relations of the structure are computable, and an equivalence relation E on $\mathbb{N}$, not necessarily computable but which is a congruence with respect to this structure, such that the quotient $\langle \mathbb{N}, \star, *, \dots \rangle$ is isomorphic to the given structure $\mathcal{A}$. Thus, one may consider computable quotient presentations of graphs, groups, orders, rings and so on. A natural question asked by B. Khoussainov in 2016, is if the Tennenbaum Thoerem extends to the context of computable presentations of nonstandard models of arithmetic. In a joint work with J.D. Hamkins we have proved that no nonstandard model of arithmetic admits a computable quotient presentation by a computably enumerable equivalence relation on the natural numbers. However, as it happens, there exists a nonstandard model of arithmetic admitting a computable quotient presentation by a co-c.e. equivalence relation. Actually, there are infinitely many of those. The idea of the proof consists is simulating the Henkin construction via finite injury priority argument. What is quite surprising, the construction works (i.e. injury lemma holds) by Hilbert's Basis Theorem. The latter argument is joint work with T. Slaman and L.

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▶ MATTIAS GRANBERG OLSSON* AND GRAHAM LEIGH, *Almost negative truth and fixpoints in intuitionistic logic.*
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We present work in progress on the relationship between the theory of transfinitely iterated strictly positive fixpoints and axiomatic theories of compositional and disquotational truth for almost negative formulae in intuitionistic logic. The starting point is the result of Cantini [1] and Feferman [2] (extended to the transfinite by Fujimoto [3]) that the (classical) theory of positive fixpoints $\widehat{\mathrm{ID}}_1$, the Kripke-Feferman compositional theory of partial truth KF, and the uniformly disquotational theory for truth-positive formulae PUTB are mutually interpretable. We obtain similar results for the theories of transfinite iterations of strictly positive fixpoints (as in [4]) for almost negative operators, and disquotation for almost negative strictly truth-positive sentences, in intuitionistic logic (which we call $\widehat{\mathrm{ID}}^{\mathrm{i}}_\alpha(\Lambda)$ and $\mathrm{P\Lambda UTB}^{\mathrm{i}}_\alpha$ respectively):

- First, $\widehat{\mathrm{ID}}^{\mathrm{i}}_\alpha(\Lambda)$ is interpretable in $\mathrm{P\Lambda UTB}^{\mathrm{i}}_\alpha$ by mimicking the classical proof.
- Second, $\mathrm{P\Lambda UTB}^{\mathrm{i}}_\alpha$ is interpretable (via a compositional theory) in $\widehat{\mathrm{ID}}^{\mathrm{i}}_{\omega\cdot\alpha}$ for limit α. This is achieved by using the extra 'spacing' between the levels, given by the multiplication by ω, to keep track of the nestings of implications in formulae.

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▶ DAICHI HAYASHI, *Reformulating supervaluational theory of truth.*
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To avoid the liar paradox, Kripke [4] used several monotone operators $\Phi : \mathcal{P}(\mathbb{N}) \to \mathcal{P}(\mathbb{N})$ that are based on partial evaluation schemata, such as the strong Kleene three-valued semantics. Using the well-known fact that such a monotone operator has fixed points, Kripke argued that sets X satisfying $\Phi(X) = X$ may be candidates for desirable extensions of the truth predicate.

As an instance of Φ, Kripke suggested a monotone operator Φ_{SV} for a supervaluation schema, to which Cantini [1] gave a corresponding axiomatic truth theory VF. While VF is satisfied in every model $\langle \mathbb{N}, X\rangle$ such that $\Phi_{SV}(X) = X$, the converse direction ($\mathbb{N}$-categoricity [2]) does not hold, that is, VF fails to completely characterise the fixed points of Φ_{SV}. Moreover, some authors have criticised VF because the axioms of VF do not mirror the structure of Φ_{SV} and thus these axioms seem somewhat unrelated.

Even worse, Fischer et al. [2] showed that no recursively enumerable first-order theory can satisfy the $\mathbb{N}$-categoricity for Φ_{SV}. Therefore, to give a supervaluation-style axiomatisation that properly mirrors the intended operator, we also need to change Φ_{SV} itself without losing its supervaluational character.

In this talk, we give another operator $\Phi_{SV'}$, which results from replacing the set-theoretic notions used in Φ_{SV} by purely semantic talk (cf. [3]). With the help of an additional typed truth predicate, an axiomatisation VF' for $\Phi_{SV'}$ is naturally induced; we show that VF' contains VF and satisfies the $\mathbb{N}$-categoricity-like result for $\Phi_{SV'}$. If time permits, we also prove that VF' has the same proof-theoretic strength as Π^1_1-CA^-.

This work was partially supported by JSPS KAKENHI, Grant Number 20J12361.

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▸ LAURI HELLA, KERKKO LUOSTO, AND JOUKO VÄÄNÄNEN*, *Dimension in team semantics*.
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We introduce three measures of complexity for families of sets. Each of the three measures, that we call dimensions, is defined in terms of the minimal number of convex subfamilies that are needed for covering the given family: for upper dimension, the subfamilies are required to contain a unique maximal set, for dual upper dimension a unique minimal set, and for cylindrical dimension both a unique maximal and a unique minimal set. In addition to considering dimensions of particular families of sets we study the behaviour of dimensions under operators that map families of sets to new families of sets. We identify natural sufficient criteria for such operators to preserve the growth class of the dimensions.

We apply the theory of our dimensions for proving new hierarchy results for logics with team semantics. First, we show that the standard logical operators preserve the growth classes of the families arising from the semantics of formulas in such logics. Second, we show that the upper dimension of $k+1$-ary dependence, inclusion, independence, anonymity, and exclusion atoms is in a strictly higher growth class than that of any k-ary atoms, whence the $k+1$-ary atoms are not definable in terms of any atoms of smaller arity.

Related and earlier work: [1][2][3][4].

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▶ ÅSA HIRVONEN AND JONI PULJUJÄRVI*, *Games and Scott sentences for positive distances between metric structures.*
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We develop an abstract framework of infinitary logic, inspired by Henson's positive bounded logic with approximations [2], and various related Ehrenfeucht–Fraïssé games that we then use to study distances defined on classes of (unbounded) metric structures. We show that the second player has a winning strategy in an EF game of length ω and precision $\varepsilon > 0$ between two separable structures exactly when the distance between the structures is $< \varepsilon$. Using tools from Scott analysis, we then obtain Scott sentences that can express the distance being $< \varepsilon$.

We study two forms of distances: pseudometrics stemming from mapping spaces onto each other with some form of approximate isomorphism, and metrics stemming from measuring the distances between two spaces isometrically embedded into a third space. Our main example of the former notion is the linear isomorphism between Banach spaces, with a fixed bi-Lipschitz constant, and of the latter notion the Kadets distance between Banach spaces.

Unlike in classical Scott analysis or Scott analysis for bounded metric structures with continuous infinitary logic [1], the traditional methods seem to only give us Scott sentences in $\mathcal{L}_{\omega_2\omega}$ rather than in $\mathcal{L}_{\omega_1\omega}$. However, if we are only concerned with distance 0, we obtain a variant of the result of [1], i.e. Scott sentences in $\mathcal{L}_{\omega_1\omega}$.

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▶ JAN HUBIČKA, *Big Ramsey degrees and trees with successor operation.*
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Recent results on big Ramsey degrees (by Dobrinen on triangle-free and Henson graphs, by Zucker on free amalgamation classes) involve formulation of special purpose tree Ramsey theorems for trees with coding nodes. The proofs of such theorems are quite involved and follow the basic scheme of the Harrington's proof of the Milliken tree theorem via the method of forcing. The main technical difficulties come from the asymmetric placement of coding nodes and complicated definitions of subtrees which need to preserve structural properties.

A recent link to the Carlson–Simpson theorem offers a new direct approach to obtaining these results. We will discuss an abstract tree theorem for trees with a sucessor operation which can be used to show all known big Ramsey degrees on binary structures and generalises to some cases of structures of higher arity. It can be seen as a joint strengthening of the Milliken tree theorem for regular trees and the Carlson–Simpson theorem for trees with unbounded branching.

This is joint work with Balko, Chodounský, Dobrinen, Konečný, Nešetřil, Vena and Zucker.

▸ ANDRZEJ INDRZEJCZAK AND PATRYCJA KUPŚ*, *Methods of modelling linear time in hypersequent calculus.*
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Linear time temporal logics have been considered in a number of proof systems, ranging from tableaux methods, natural deduction and sequent calculi. However, most of these approaches are focused on the extensions of Prior's tense logic (TL). We are particularly interested in systems that model Linear Time Logic (LTL), which extend TL by *Next Time* and *Until* operators. We also choose to focus on the hypersequent calculus (HC) since it proved to increase the expressive power of the sequent calculus in temporal logics. We offer a review of already existing methods of modelling linear time in a hypersequent approach and outline the possibility of providing an efficient proof system for LTL in HC.

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▸ MAIRA KASSYMETOVA, NAZGUL SHAMATAYEVA, OLGA ULBRIKHT* AND AIBAT YESHKEYEV, *Properties of lattices of existential formulas of Jonsson beautiful pairs.*
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Let T be a convex $\exists$-complete perfect Jonsson theory of a countable language L, C is its semantic model, $T^* = Th(C)$ is the center of theory T, $M = \bigcap M_i$, where $M_i \in E_T$, $M_i \subseteq C$ and E_T is the class of existentially closed models of the theory T.

Definition 1. Let $N, M \in E_T$ and $M \preceq_{\exists_1} N$. We will call a pair (N, M) a J-beautiful pair if it satisfies the following conditions:
1) M is $|T|^+$-$\exists_1$-saturated;
2) for each tuple $\bar{b}$ extracted from N, each $\exists$-type over $M \cup \{\bar{b}\}$ is realized in N.

Let class $K = \{(M_i, M) \mid M_i \preceq_{\exists_1} C$ and (M_i, M) is J-beautiful pair$\}$. Consider the Jonsson spectrum of the class K:

$$JSp(K) = \{\Delta \mid \Delta \text{ is Jonsson theory and } \Delta = Th_{\forall\exists}(M_i, M), \text{ where } (M_i, M) \in K\}.$$

It is easy to see that the cosemanticness relation on the set of Jonsson theories is an equivalence relation. Then we can consider the $JSp(K)/_{\bowtie}$, which is the factor set of the Jonsson spectrum of the class K with respect to $\bowtie$.

Let $[\Delta] \in JSp(K)/_{\bowtie}$ and $E_n([\Delta])$ be the distributive lattice of equivalence classes of $\varphi^{[\Delta]} = \{\psi \in E_n(L) \mid [\Delta]^* \models \varphi \leftrightarrow \psi,\ \varphi \in E_n(L)\}$.

Definition 2. Let T be an arbitrary Jonsson theory, then a $\sharp$-companion of a theory T is a theory $T^\sharp$ of the same signature if it satisfies the following conditions:
(i) $(T^\sharp)_\forall = T_\forall$;
(ii) if $T_\forall = T'_\forall$, then $T^\sharp = (T')^\sharp$;
(iii) $T_{\forall\exists} \subseteq T^\sharp$.

The natural interpretations of the companion $T^\sharp$ are T^*, T^f, T^M, T^e, T^0, where T^* is the center of Jonsson theory T, T^f is the forcing companion of Jonsson theory T, T^M is the model companion of the theory T, $T^e = Th(E_T)$, $T^0 = Th_{\forall\exists}C$.

Theorem 3. *Let $[\Delta] \in JSp(K)/_{\bowtie}$ be a perfect class, then the following conditions are equivalent:*
(i) $[\Delta]^\sharp$ is complete theory;
(ii) $[\Delta]^\sharp$ is model complete theory.

Theorem 4. *Let $[\Delta] \in JSp(K)/_{\bowtie}$ be a complete for $\exists$-sentences class. Then the following conditions are equivalent:*
(i) $[\Delta]$ is perfect;
(ii) $[\Delta]^*$ is model-complete theory;
(iii) $E_n([\Delta])$ is a Boolean algebra.

Theorem 5. *Let $[\Delta] \in JSp(K)/_{\bowtie}$ be a perfect $\forall\exists$-complete convex class, $[\Delta]^\sharp$ be its $\sharp$-companion. Then theory $[\Delta]^\sharp$ is ω-categorical iff the class $[\Delta]$ is ω-categorical.*

All necessary concepts that are not defined in this thesis can be extracted from [1].

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP09260237).

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▶ PETER KOEPKE, *Formalizing the Appendix of Kelley's General Topology in the Naproche Proof Checker.*
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The $\mathbb{N}$aproche natural language proof checker allows proof checked mathematical formalizations that read like ordinary mathematical texts [2, 3]. The easiest way to use the $\mathbb{N}$aproche system is to install the well-known Isabelle prover [4] and to edit a file with a `.ftl.tex` suffix in the Isabelle/jEdit editor. Some example files are included with the Isabelle distribution.

In my talk I shall describe a $\mathbb{N}$aproche formalization of the appendix of John L. Kelley's *General Topology* [1]. The appendix is a short introduction to Kelley-Morse set theory which is taken as the foundation for the book and which is developed up to some initial results on ordinals and cardinals. Kelley's text lends itself to formalizations due to its formalistic and complete style of writing. Indeed many statements of the appendix can be transferred almost verbatim to $\mathbb{N}$aproche. Considering, e.g., Kelley's

38 THEOREM *If x is a set, then 2^x is a set, and for each y, $y \subset x$ iff $y \varepsilon 2^x$.*

about the power class 2^x of x, the obvious LaTeX source is a legitimate $\mathbb{N}$aproche input, save for the commas:

```
\begin{theorem}
If $x$ is a set then $2^x$ is a set and for each $y$ $y \subset x$
iff $y \in 2^x$.
\end{theorem}
```

Based on the Kelley formalization I shall conclude with general remarks on the natural language of mathematics.

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Study of big Ramsey degrees is an infinitary extension of the study of Ramsey classes. While being stated in a purely combinatorial manner, it is closely connected to model theory (the objects of study are homogeneous structures), topological dynamics (its results are used to construct *universal completion flows* of automorphism groups) and set theory (the tools used are infinitary tree Ramsey theorems such as the Milliken theorem or the Carlson–Simpson theorem, as well as Harrington's application of the method of forcing).

It turns out that *trees of types* are fundamental for understanding big Ramsey degrees. Given an enumeration of an ultrahomogeneous structure, the tree of n-types is formed by realised n-types over finite initial segments of the enumeration. For structures in binary languages, these trees have bounded branching, but this fails with relations of arity at least three, which makes the problems significantly more complicated.

Recently, we were able to show that an unconstrained homogeneous relational structure has finite big Ramsey degrees if and only if it is ω-categorical (if and only if its tree of n-types is finitely branching for every n). In particular, this is the first time we were able to handle structures in infinite languages.

This is joint work with Samuel Braunfeld, David Chodounský, Noé de Rancourt, Jan Hubička and Jamal Kawach.

▸ KRZYSZTOF KRUPIŃSKI AND ADRIÁN PORTILLO*, *On stable quotients.*
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We solve two problems from Haskel and Pillay [1], which concern maximal stable quotients of groups type-definable in NIP theories. The first result says that if G is a type-definable group in a distal theory, then $G^{st} = G^{00}$ (where G^{st} is the smallest type-definable subgroup with G/G^{st} stable, and G^{00} is the smallest type-definable subgroup of bounded index). In order to get it, we prove that distality is preserved under passing from T to the hyperimaginary expansion T^{heq}. The second result is an example of a group G definable in a non-distal, NIP theory for which $G = G^{00}$ but G^{st} is not an intersection of definable groups. Our example is a saturated extension of $(\mathbb{R}, +, [0, 1])$. Moreover, we make some observations on the question whether there is such an example which is a group of finite exponent. We also take the opportunity and give several characterizations of stability of hyperdefinable sets, involving continuous logic.

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▶ BEIBUT KULPESHOV AND SERGEY SUDOPLATOV*, *On theories of dense spherical orders.*
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We study properties of theories of n-spherical orders K_n [1, 2] which naturally generalize linear orders K_2 and circular orders K_3 [3, 4, 5].

A *n-spherical* order relation, for $n \geq 2$, is described by a n-ary relation K_n satisfying the following conditions: (nso1) $\forall x_1, \ldots, x_n(K_n(x_1, x_2, \ldots, x_n) \to K_n(x_2, \ldots, x_n, x_1))$; (nso2) $\forall x_1, \ldots, x_n \Bigg((K_n(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n) \wedge K_n(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)) \leftrightarrow \bigvee\limits_{1\leq k<l\leq n} x_k \approx x_l \Bigg)$ for any $1 \leq i < j \leq n$; (nso3) $\forall x_1, \ldots, x_n \Bigg(K_n(x_1, \ldots, x_n) \to \forall t \left(\bigvee\limits_{i=1}^{n} K_n(x_1, \ldots, x_{i-1}, t, x_{i+1}, \ldots, x_n) \right) \Bigg)$; (nso4) $\forall x_1, \ldots, x_n(K_n(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n) \vee \vee K_n(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n))$, $1 \leq i < j \leq n$.

Structures $\mathcal{A} = \langle A, K_n \rangle$ with n-spherical orders are called *n-spherical orders*, too.

A n-spherical order K_n is called *dense* if it contains at least two elements and for each $(a_1, a_2, a_3, \ldots, a_n) \in K_n$ with $a_1 \neq a_2$ there is $b \notin \{a_1, a_2, \ldots, a_n\}$ with $\models K_n(a_1, b, a_3, \ldots, a_n) \wedge K_n(b, a_2, a_3, \ldots, a_n)$.

Theorem 1. *If $\mathcal{A}$ and $\mathcal{B}$ are countable dense n-spherical orders, $n \geq 2$, without endpoints for $n = 2$, then $\mathcal{A} \simeq \mathcal{B}$.*

Theorem 2. *For any natural $n \geq 2$ the theory T_n of dense n-spherical order is decidable.*

The research is supported by Committee of Science in Education and Science Ministry of the Republic of Kazakhstan, Grant No. AP08855544 and Russian Scientific Foundation, Project No. 22-21-00044. The work of the second author was carried out in the framework of the State Contract of the Sobolev Institute of Mathematics, Project No. FWNF-2022-0012.

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▶ BEIBUT KULPESHOV* AND SERGEY SUDOPLATOV, *On algebras of binary formulas for weakly circularly minimal theories.*
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Algebras of binary formulas are a tool for describing relationships between elements of the sets of realizations of 1-types at binary level with respect to superpositions of binary definable sets. We consider algebras of binary isolating formulas originally studied in [1, 2], where under a binary isolating formula we understand a formula of the form $\varphi(x, y)$, without parameters, such that for some parameter a the formula $\varphi(a, y)$ isolates some complete type from $S_1(\{a\})$.

The notion of *weak circular minimality* was originally studied in [3]. A *weakly circularly minimal structure* is a circularly ordered structure $M = \langle M, K_3, \dots \rangle$ such that any definable (with parameters) subset of M is a union of finitely many convex sets in M. In [4] countably categorical 1-transitive non-primitive weakly circularly minimal structures of convexity rank 1 with non-trivial definable closure have been described up to binarity.

Here we discuss algebras of binary isolating formulas for these structures and give the following criterion for commutability of such algebras:

Theorem 1. *Let M be a countably categorical 1-transitive non-primitive weakly circularly minimal structure of convexity rank 1 with $\mathrm{dcl}(a) \neq \{a\}$ for some $a \in M$. Then the algebra $\mathfrak{P}_M$ of binary isolating formulas is commutable iff there exists an $\emptyset$-definable non-trivial monotonic-to-right bijection on M.*

This research has been funded by Science Committee of Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08855544), and by Russian Scientific Foundation (Project No. 22-21-00044).

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▶ ELIO LA ROSA, *Normalization of epsilon calculus.*
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Rule-based reformulations of Hilbert and Bernays' epsilon calculus have been attempted in an effort to provide Gentzen-style normalization procedures. The problem, however, is an open one [3]. In this contribution, a normalization procedure is developed for a system of epsilon calculus based on a structural extension of natural deduction allowing for multiple conclusions. The base propositional system has explicit structural rules and introduction and elimination rules that are both local and 'symmetric', in the sense that they do not depend on assumptions and are allowed to branch the derivation upwards and downwards, respectively. The somewhat complicated structure of the derivations in the system, similar to that of [2], is mitigated by the fact that branching and rule applications can be reinterpreted in a system of 'open deduction' [1]. The rules introducing epsilon terms are a simple rule reformulation of the "critical formulas" found in the axiomatic version of epsilon calculus. A normalization procedure is easy to formulate for the propositional part, which allows for a first elimination of detours in the derivation. For what concerns detours caused by applications of rules introducing epsilon terms which do not appear as premises or conclusions in the derivation, reductions based on replacement of epsilon terms indexed by rule instances are defined. The procedure yields the first epsilon theorem and provides a form of the subformula property for the calculus.

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▸ MATEUSZ ŁEŁYK* AND BARTOSZ WCISŁO, *Model-theoretical characterizations of truth*.
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The talk is devoted to the exposition and explanation of the recent results on model-theoretical characterizations of axiomatic theories of various semantical notions. We consider the axiomatic theories of truth, definability and satisfaction, which are well-known to have interesting model theoretical features. For example, the minimal axiomatic theory of truth for a language $\mathcal{L}$, $\mathrm{TB}^-(\mathcal{L})$, uniformly imposes elementarity, in the sense that for any two models $\mathcal{M},\mathcal{N} \models \mathrm{TB}^-(\mathcal{L})$, if $\mathcal{M}$ is a submodel of $\mathcal{N}$, then their $\mathcal{L}$-reducts are elementarily equivalent (have the same $\mathcal{L}$-theory). Moreover, in the presence of induction axioms for the satisfaction predicate, the minimal theory of satisfaction for the language $\mathcal{L}$, $\mathrm{USB}(\mathcal{L})$, imposes $\mathcal{L}$-recursive saturation, in the sense that for any model $\mathcal{M} \models \mathrm{USB}(\mathcal{L})$, the $\mathcal{L}$-reduct of $\mathcal{M}$ is recursively saturated. For every such model-theoretical property we ask whether it characterizes the respective theory up to definability.

Following [2], we work in a general context of sequential theories, and for an arbitrary such theory U and a recursive language $\mathcal{L}$ we define our target axiomatic theory

$\mathrm{TB}^-(\mathcal{L})$ extends U with all axioms of the form $T(\ulcorner\phi\urcorner) \equiv \phi$.
$\mathrm{DEF}^-(\mathcal{L})$ extends U with all axioms of the form $\forall y\big(D(\ulcorner\phi(x)\urcorner, y) \equiv \exists! x\phi(x) \wedge \phi(y)\big)$.
$\mathrm{USB}^-(\mathcal{L})$ extends U with all axioms of the form $\forall x\big(S(\ulcorner\phi(x)\urcorner, x) \equiv \phi(x)\big)$.

In the above T, D, S are fresh predicates and $\phi, \phi(x)$ range over $\mathcal{L}$ sentences and formulae respectively. In each case, the Gödel codes of sentences come from a fixed interpretation of the Buss's theory S^1_2 in U. As in the case of $\mathrm{TB}^-(\mathcal{L})$ each of the above theories is associated with a model-theoretical property: $\mathrm{DEF}^-(\mathcal{L})$ can be shown to uniformly preserve $\mathcal{L}$-definability and $\mathrm{USB}^-(\mathcal{L})$ – to uniformly impose elementarity. We show that each of the above theories is minimal w.r.t. definability among theories which have the respective property. In particular, in the context of $\mathrm{TB}^-(\mathcal{L})$ this means that any r.e. sequential theory in a finite language which uniformly imposes $\mathcal{L}$-elementary equivalence, syntactically defines $\mathrm{TB}^-(\mathcal{L})$. As a byproduct, we answer an open problem from [2], showing that $\mathrm{TB}^-(\mathcal{L})$ does not admit a restricted axiomatization (similarly for $\mathrm{DEF}^-(\mathcal{L})$).

Moreover, we study possible strengthenings of Kossak's theorem [1], saying that if a theory U (in at most countable language) extends PA, proves all induction axioms for formulae of $\mathcal{L}_U$ and imposes $\mathcal{L}_{\mathrm{PA}}$-recursive saturation, then in every model of U

USB($\mathcal{L}_{\rm PA}$) is definable with parameters. We show that a variant of this result holds for all theories (in at most countable language) which extend PA: if U is such a theory and U imposes $\mathcal{L}_{\rm PA}$-recursive saturation, then USB$^-$($\mathcal{L}_{\rm PA}$) is definable in every model of U. Finally, we show that Kossak's theorem cannot be strengthened by requiring that USB($\mathcal{L}_{\rm PA}$) is syntactically definable in U: there is a theory U which satisfies the assumptions of Kossak's theorem but no $\mathcal{L}_U$-formula defines the satisfaction predicate across all models of U.

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▶ IOANA LEUŞTEAN AND BOGDAN MACOVEI*, *Fully Certified Dynamic Epistemic Logic for Security Protocols.*

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The formal analysis of security protocols is a challenging field, with various approaches being studied nowadays. The famous Burrows-Abadi-Needham Logic was the first logical system aiming to validate security protocols. Combining ideas from previous approaches, in this paper we define a complete system of *dynamic epistemic logic* for modeling security protocols. Our logic is implemented and fully verified, using theorem prover Lean.

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▶ MAXWELL LEVINE, *Unthreadability at $\aleph_2$.*

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The tension between the stationary reflection principles of large cardinals and the fine-structural combinatorics that hold in canonical inner models is a prominent theme in set theory. This program of research helps us develop a more vivid picture of the individual cardinals and their relationships to one another. In particular, variations of Jensen's square principle $\square_\kappa$, which holds for all cardinals κ in Gödel's Constructible

Universe L, have been studied widely. In this talk, we will introduce a new partial order that is countably closed and adds a $\square(\aleph_2, \aleph_0)$-sequence with countable conditions. This differs significantly with the known methods for adding square sequences.

▸ JOSÉ M. MÉNDEZ*, GEMMA ROBLES AND FRANCISCO SALTO, *A class of implicative expansions of Belnap-Dunn logic in whose elements a Boolean negation is definable*.
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Let B′ be the result of restricting Routley and Meyer basic relevant logic B (cf. [3]) as follows: (1) Restrict all axioms of B to rule form, except the self-identity axiom (i.e., $A \to A$), the distributive and the double negation axioms. (2) Restrict the rules Suffixing and Prefixing to the rule Transitivity. Then, we define the class of all C-extending implicative expansions containing B′ of the well-known Belnap-Dunn logic in whose elements a Boolean negation is definable. We note that, apart from classical logic, in each one of these expansions the strong logic PŁ4 is definable. PŁ4 is equivalent to De and Omori's logic BD_+, Zaitzev's paraconsistent logic FDEP and Béziau's 4-valued modal logic PM4N, according to [1] (cf. [2] and references therein).

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▸ ANTONIO MONTALBÁN AND DINO ROSSEGGER*, *The structural complexity of models of arithmetic*.
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The Scott rank of a countable structure is the least ordinal α such that all automorphism orbits of the structure are definable by infinitary Σ_α formulas. Montalbán showed that the Scott rank of a structure is a robust measure of its structural and computational complexity by showing that various different measures are equivalent. For example, a structure has Scott rank α if and only if it has a $\Pi_{\alpha+1}$ Scott sentence if and only if it is uniformly $\mathbf{\Delta}^0_\alpha$ categorical. In this talk we present results on the Scott rank of models of Peano arithmetic. We show that non-standard models of PA have Scott rank at least ω and that the models of PA that have Scott rank ω are precisely the prime models. We also give reductions via bi-interpretability of the class of linear orders to completions T of PA. This allows us to exhibit models of T of Scott rank α for every $\omega \leq \alpha \leq \omega_1$.

▸ NAZERKE MUSSINA*, OLGA ULBRIKHT AND AIBAT YESHKEYEV, *Syntactic and semantic similarities of hybrids of classes of the Jonsson spectrum of Jonsson quasivariety of the class K.*
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Let K be the class of structures of countable signature σ. Let's introduce the notation:

$$\forall\exists(K) = Th(K) \cup \{\varphi \mid \varphi \text{ is a } \forall\exists\text{-sentence of considered language and } \varphi \cup Th(K) \text{ is a consistent}\}.$$

Definition 1. A variety (quasivariety) of structures K is called a Jonsson variety (quasivariety) if $\forall\exists(K)$ is a Jonsson theory.

Consider the $JSpV(K)$ be Jonsson spectrum of the Jonsson variety of class K, where K is the Jonsson variety:

$$JSpV(K) = \{T \mid T = \forall\exists(N) \text{ is Jonsson theory, } N \text{ is a subvariety of } K\}.$$

Then $JSpV(K)/_{\bowtie}$ is denoting the factor set of the Jonsson spectrum of Jonsson quasivariety of the class K by the relation $\bowtie$.

Similarly, we define the Jonsson spectrum of $JSpQV(K)$ quasivariety:

$$JSpQV(K) = \{T \mid T = \forall\exists(N) \text{ is Jonsson theory, } N \text{ is a subquasivariety of } K\}.$$

Then $JSpQV(K)/_{\bowtie}$ denotes the factor set of the Jonsson spectrum of Jonsson quasivariety of the class K by the relation $\bowtie$.

Definition 2. Let K be some Jonsson quasivariety of structures of signature σ, $[T_1], [T_2] \in JSpQV(K)/_{\bowtie}$. The hybrid (of the first type) $H([T_1], [T_2])$ of the classes $[T_1]$ and $[T_2]$ is the theory $Th_{\forall\exists}(C_1 \diamond C_2)$ if it is Jonsson theory in language of the signature σ, where C_i are semantic models of the classes $[T_i]$, $i = 1, 2$ respectively and $\diamond \in \{\times, +, \oplus, \prod_F, \prod_U\}$, where $\times$ is cartesian product, $+$ is the sum, $\oplus$ is the direct sum, $\prod_F$ is reduced product and $\prod_U$ is the ultraproduct of models.

The following fact will be necessary for the proof of Theorem 1.

Fact 1. ([1], p. 48) For any complete for $\exists$-sentences Jonsson theory T the following conditions are equivalent:

1) T^* is model complete;

2) for each $n < \omega$, $E_n(T)$ is Boolean algebra, where $E_n(T)$ is a lattice of $\exists$-formulas with n free variables.

And in the frame above mentioned notions we have the following result.

Theorem 1. *Let K be some Jonsson quasivariety of structures of signature σ, $[T_1], [T_2], [T_3], [T_4] \in JSpQV(K)/_{\bowtie}$, $H_1 = H([T_1], [T_2])$ and $H_2 = H([T_3], [T_4])$ are complete for existential sentences perfect hybrids, then following conditions are equivalent:*

1. $H_1 \overset{S}{\sim} H_2$;

2. $H_1^* \overset{S}{\approx} H_2^*$.

All additional information regarding Jonsson theories can be found in [1].

This work was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (grand AP09260237).

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▶ HRAFN ODDSSON, *Paraconsistent and paracomplete Zermelo-Fraenkel set theory.*
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This work is joint with Yurii Khomskii. We present a treatment of set theory in a four-valued paracomplete and paraconsistent logic, i.e., a logic in which propositions can be neither true nor false, and can be both true and false. Our approach differs from most previous attempts since we are not interested in satisfying full comprehension or avoiding Russell's paradox. Rather, we prioritise setting up a system with a clear ontology of non-classical sets, which can be used to reason informally about incomplete and inconsistent phenomena.

We propose an axiomatic system BZFC, obtained by carefully analysing the ZFC-axioms and transferring the axioms appropriately. Moreover, we introduce the anti-classicality axiom postulating the existence of non-classical sets, and prove a surprising result stating that the existence of a single non-classical set is sufficient to produce any other type of non-classical set.

We also look at natural bi-interpretability results between BZFC and classical ZFC.

▶ LUIZ CARLOS PEREIRA AND ELAINE PIMENTEL, *On an ecumenical natural deduction with* stoup.
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Natural deduction systems, as proposed by Gentzen [1] and further studied by Prawitz [3], is one of the most well known proof-theoretical frameworks. Part of its success is based on the fact that natural deduction rules present a simple characterization of logical constants, especially in the case of intuitionistic logic. However, there has been a lot of criticism on extensions of the intuitionistic set of rules in order to deal with classical logic. Indeed, most of such extensions add, to the usual introduction and elimination rules, extra rules governing negation. As a consequence, several meta-logical properties, the most prominent one being *harmony*, are lost.

In [4], Dag Prawitz proposed a natural deduction *ecumenical system*, where classical logic and intuitionistic logic are codified in the same system. In this system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation and the constant for the absurd, but they would each have their own existential quantifier, disjunction and implication, with different meanings. Prawitz' main idea is that these different meanings are given by a semantical framework that can be accepted by both parties.

In this talk, we propose a different approach adapting, to the natural deduction framework, Girard's mechanism of *stoup* [2]. This will allow the definition of a pure harmonic natural deduction system ($\mathcal{LE}_p$) for the propositional fragment of Prawitz' ecumenical logic.

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▸ IOSIF PETRAKIS, *Positive negation in constructive mathematics.*
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In standard constructive logic negation is treated as in classical logic in a negativistic and weak way. This is in contrast to the use of a positive and strong "or" and "exists". In constructive mathematics [1] however, we often find a positive and strong approach to negatively defined concepts, like that of inequality. This fact motivates a clear distinction between a positive and strong negation and the standard weak negation. Bringing together older ideas of Griss and Nelson and recent work of Shulman [3] and ours [2], we investigate the role of a positive and strong negation in Bishop-style constructive mathematics BISH. We define the positive negation of a formula in BISH, we determine the formulas of BISH that are used to define the equality of a Bishop set, and we define the canonical inequality of a Bishop set through positive negation of its given equality formula. Consequently, many seemingly ad hoc definitions of concepts of BISH, such as the complement of a subset, the empty subset, complemented subsets, and the F-complement of a closed set, are canonical definitions through positive negation.

[1] E. Bishop, D. Bridges, ***Constructive Analysis***, Springer-Verlag, 1985.
[2] I. Petrakis, *Families of Sets in Bishop Set Theory*, ***arXiv:2109.04183v1*** (2021).
[3] M. Shulman, *Affine Logic for Constructive Mathematics*, ***arXiv:1805.07518v2*** (2021).

▸ PHILIPP PROVENZANO, *The reverse mathematical strength of hyperations.*
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Hyperations have been introduced in [1] as a way to transfinitely iterate normal, i.e., strictly increasing continuous, functions on ordinals, refining the notion of Veblen functions. The goal of this talk is to outline a construction of hyperations for certain uniform functions on linear orders in second order arithmetic and discuss the logical strength of preservation of well-foundedness by this construction in the light of reverse mathematics.

For the first goal, we will employ the notion of dilators introduced by Girard and show how a sufficiently uniform categorical treatment of finite iterations can be extended to transfinite exponents. Such a construction has already appeared for the standard Veblen hierarchy in [2].

The proof-theoretic discussion builds on a framework developed in [3], relating transfinitely iterated syntactic reflection to semantic ω-model reflection. The ordinal analysis of ATR_0 developed there is relativized to an arbitrary normal dilator T, yielding an equivalence between the principles "*the hyperation of T preserves well-foundedness*" and $\Pi^1_2\text{-}\omega\,\mathrm{RFN}(\Pi^1_1\text{-}\,\mathrm{BI}_0 + T$ *is a dilator*) over the weak base theory RCA_0.

The master thesis on which this talk is based has been supervised by Andreas Weiermann and Fedor Pakhomov from the Logic group at Ghent University.

[1] David Fernández-Duque and Joost J. Joosten, *Hyperations, Veblen Progressions and Transfinite Iteration of Ordinal Functions*, ***Annals of Pure and Applied Logic***, vol. 164 (2013), no. 7-8, pp. 785–801.

[2] Jean-Yves Girard and Jacqueline Vauzeilles, *Functors and ordinal notations. I: A functorial construction of the veblen hierarchy*, ***Journal of Symbolic Logic***, vol. 49 (1984), no. 3, pp. 713–729.

[3] Fedor Pakhomov and James Walsh, *Reducing ω-model reflection to iterated syntactic reflection*, ***Journal of Mathematical Logic***, 2021, doi: 10.1142/S0219061322500015

▸ JONI PULJUJÄRVI AND DAVIDE EMILIO QUADRELLARO*, *Compactness and Types in Logics of Dependence.*
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In first-order logic, the following formulations of the compactness theorem can be easily proved from one another:

(i) *Every set of sentences that is finitely satisfiable is satisfiable*;
(ii) *Every set of formulas that is finitely satisfiable is satisfiable.*

For dependence logic, the first version of compactness is a well-known result and was proved by Väänänen in [1] using the translation between dependence logic and Σ^1_1. However, in the context of dependence logic, one cannot derive (ii) from (i) by replacing variables with constants, as it is the case for first order logic.

The second version of compactness (ii) has been recently considered by Kontinen and Yang in [2], who used the translation from dependence logic to Σ^1_1 to show that "*every set of formulas with countably many free variables that is finitely satisfiable is satisfiable*". In our talk, we provide a proof of the second version of compactness (ii) for arbitrary sets of formulas by adapting ultraproducts to the context of team semantics, analogously to [3].

Finally, we briefly touch upon the issue of types in dependence logic, and we see how to obtain a compact space of suitable type.

[1] Jouko Väänänen, *Dependence Logic: A New Approach to Independence Friendly Logic*, Cambridge University Press, 2007.

[2] Juha Kontinen and Fan Yang, *Complete logics for elementary team properties*, https://arxiv.org/abs/1904.08695.

[3] Martin Lück, *Team Logic Axioms, Expressiveness, Complexity*, PhD thesis, University of Hannover, 2020.

▸ ALEXEJ PYNKO, *Minimally n-valued maximally paraconsistent expansions of LP*.
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Given any propositional language L (viz., a set of propositional connectives, treated as operation symbols, when dealing with L-algebras), a propositional L-logic C (viz., a structural closure operator over the carrier Fm_L of the absolutely-free L-algebra $\mathfrak{Fm}_L$ freely-generated by the set $V \triangleq \{x_i\}_{i\in\omega}$ of propositional variables $\langle$as usual, natural numbers, including 0, are treated as sets of lesser ones, the set of all them being denoted by $\omega\rangle$) is said to be *[{uniformly/axiomatically} minimally/maximally] "⌊singularly⌋ ⌈no-more-than-⌉n-valued"/¬-paraconsisent*, where "$n \in (\omega \setminus (1[+1])$"/"$\neg \in L$ is unary", provided "C is *defined* by a ⌊one-element⌋ class M of ⌈no-more-than-⌉n-valued L-*matrices* (viz., pairs of L-algebras and their subsets) — i.e., $\{h^{-1}[D] \mid \langle \mathfrak{A}, D\rangle \in \mathsf{M}, h \in \mathrm{hom}(\mathfrak{Fm}_L, \mathfrak{A})\}$ is a closure basis of img$C$ — [but is not {singularly} no-more-than-$(n-1)$-valued]"/"$x_1 \notin C(\{x_0, \neg x_0\})$ [and C has no ¬-paraconsistent *extension* C' (viz., an L-logic with $(\mathrm{img}C') \subseteq (\mathrm{img}C)$) such that $C'\{(\varnothing)\} \neq C\{(\varnothing)\}$]", an L-matrix being said to be $\neg$-*paraconsistent*, whenever its (viz., defined by it) logic is so. Then, a *model of* C is any L-matrix defining an extension of C.

Let $n \in (\omega \setminus 3)$, $L_{+[-]} \triangleq \{\wedge, \vee[, \neg]\}$ the propositional language with binary connectives [other than the unary one $\neg$], $N_{n[-]} \triangleq \{i \in ((n-1)\setminus 1) \mid (2\cdot i) \in (n[-1]) \ni (4[-3])\}$, $L_n \triangleq (L_{+-} \cup \{\partial_i \mid i \in N_{n-}\} \cup \{\nabla_j \mid j \in N_n\})$ the propositional language with unary connectives other than those in L_+, $\mathfrak{A}_n$ the L_n-algebra with L_{+-}-reduct being the Kleene chain lattice under the natural ordering on the carrier n of $\mathfrak{A}_n$ as well as operations $\partial_i^{\mathfrak{A}_n} \triangleq (((i+1) \times \{0\}) \cup ((n \setminus (i+1)) \times \{n-1\}))$, where $i \in N_{-n}$, and $\nabla_j^{\mathfrak{A}_n} \triangleq ((((n-1) \setminus 1) \times \{j\}) \cup \{\langle 0,0\rangle, \langle n-1, n-1\rangle\})$, where $j \in N_n$, while $\mathcal{A}_n \triangleq \langle \mathfrak{A}_n, D_n\rangle$ the L_n-matrix with $D_n \triangleq (n \setminus 1)$, whereas C_n the logic of $\mathcal{A}_n$. in which case this is $\neg$-paraconsistent, while $(L|C)_3 = (L_{+-}|LP)$ |(viz., the *logic of paradox*), whereas $((((n-1) \setminus 1) \times \{1\}) \cup \{\langle 0,0\rangle, \langle n-1,n-1\rangle\}) \in \hom(\mathcal{A}_n \restriction L_3, \mathcal{A}_3)$ is both strict and surjective, and so C_n is an n-valued expansion of LP (in particular, LP is [non-minimally] n-valued [unless $n = 3$], the L_+-fragment of C_n being that of LP {i.e., that of PC}).

Lemma 1. *For any $\neg$-paraconsistent model $\langle \mathfrak{A}, D\rangle$ of C_n, there are some subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ with carrier B and some surjective $h \in \hom(\mathfrak{B}, \mathfrak{A}_n)$ with $(B \cap D) = h^{-1}[D_n]$.*

As any L-logic is defined by the class of all its models, Lemma 1 immediately yields:

Theorem 2. *C_n is both minimally n-valued and maximally $\neg$-paraconsistent.*

On the other hand, elimination of any connective in $L_n \setminus L_{+-}$ results in a fragment of C_n that is either not (even uniformly) minimally n-valued or not (even axiomatically) maximally $\neg$-paraconsistent. More precisely, we have:

Theorem 3. *The L'-fragment C' of C_n with $L' \subseteq | = (L_n \setminus \{(\partial/\nabla)_i\})$, $i \in N_{(n-)|n}$, is not uniformly|axiomatically minimally|maximally n-valued|$\neg$-paraconsistent.*

Theorem 4. *Let $C_n^{\mathrm{NP/PC}}$ be the L_n-logic defined by "the direct product of $\mathcal{A}_n$ and"/ $\langle \mathfrak{A}_n \restriction \{0, n-1\}, \{n-1\}\rangle$. Then, these are the only proper $\lceil$viz., distinct from $C_n\rceil$ consistent $\lfloor$viz., not defined by $\varnothing\rfloor$ extensions of C_n, while the former/latter is /"a proper extension of the former as well as" the least non-$\neg$-paraconsistent/ extension /C' of C_n /"such that $(x_1[|\neg x_0 \supset (x_0 \supset x_1)]) \in C'(\{x_0\{\vee x_1\}, \neg x_0 \vee x_1\}[|\varnothing])$ [whenever $4 \in n$, where $(x_0 \supset x_1) \triangleq (\partial_1 \nabla_1 \neg x_0 \vee x_1)$]", whereas $C_n^{\mathrm{NP}}(\varnothing) = C_n(\varnothing)(= C_n^{\mathrm{PC}}(\varnothing)$ iff $4 \notin n$).*

▸ ALEXEJ PYNKO* AND GNAT RUBKO, *Paraconsistent extensions of three-valued logics.*
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Given any propositional language L (viz., a set of connectives, treated as operation symbols, when dealing with L-algebras), an *L-logic* C (viz., a structural closure operator over the set Fm_L of L-formulas with variables in $\{x_i\}_{i\in\omega}$ $\langle$natural numbers, including 0, are treated as sets of lesser ones, the set of all them being denoted by $\omega\rangle$; pairs of the form $\Gamma \vdash \varphi$, where $\Gamma \subseteq \mathrm{Fm}_L \ni \varphi$, being called *$L$-rules*) is said to "*satisfy* an L-rule $\Gamma \vdash \varphi$"/"be *[(\{almost\} pre-)maximally]* $\langle\neg$-*para*$\rangle$*consisent* $\langle$where $\neg \in L$ is unary$\rangle$", if "$\varphi \in C(\Gamma)$"/"C does not satisfy $(\langle\{x_0, \neg x_0\}\cup\rangle\varnothing) \vdash x_1$ [and has no (more than 1\{+1\}) $\langle\neg$-para$\rangle$consistent *proper* — viz., distinct from C – *extension* — viz., an L-logic satisfying all L-rules C satisfies], an L-matrix defining such a logic being said to do/be so too". Likewise, C is said to be *weakly $\wedge$-conjunctive/$\vee$-disjunctive*, where $\wedge/\vee$ is a binary connective of L (possibly, a *secondary* one; viz., an L-formula with *at most* two variables x_0 and x_1), if $C(x_{0|1}) \subseteq / \supseteq C(x_0(\wedge/\vee)x_1)$, an

L-matrix defining such a logic being said to be so too. Then, a *theorem/model of* C is any "element of $C(\varnothing)$"/"L-matrix defining an extension of C". Likewise, the least extension C^R of C satisfying an L-rule R is said to be *relatively axiomatized by* R. Finally, two-valued L-matrices with single distinguished value and operation $\neg$ permuting their unique distinguished and non-distinguished values are said to be *$\neg$-classical*, [any *sublogic of* — viz., an L-logic with an extension being — any of] their logics being called *$\neg$-[sub]classical*. Let $\mathcal{A} = \langle \mathfrak{A}, D \rangle$ be a $\neg$-paraconsistent L-matrix with carrier $A \triangleq (2 \cup \{\frac{1}{2}\})$ and $(\neg^{\mathfrak{A}} \upharpoonright 2) = (2^2 \setminus \Delta_2)$, where $\Delta_S \triangleq \{\langle s, s \rangle \mid s \in S\}$, for any set S, as well as $D \triangleq (A \setminus 1)$, $\mathcal{A}_{\frac{1}{2}} \triangleq \langle \mathfrak{A}, \{\frac{1}{2}\} \rangle$ and $C_{[\frac{1}{2}]}$ the logic of $\mathcal{A}_{[\frac{1}{2}]}$.

THEOREM 1. *C is not maximally $\neg$-paraconsistent iff $(\neg^{\mathfrak{A}} \cup \{\langle \frac{1}{2}, \frac{1}{2} \rangle\}) \in \mathbf{S}\mathfrak{A}^2$ iff $\neg^{\mathfrak{A}}$ is an automorphism of $\mathfrak{A}$ iff $(C/\mathcal{A})_{\frac{1}{2}}$ is a $\neg$-paraconsistent extension/model of C iff C has a $\neg$-paraconsistent model with single distinguished value iff $\langle \mathfrak{A}, \{0, \frac{1}{2}\} \rangle$ is a $\neg$-paraconsistent/defining model of C iff the extension of C relatively axiomatized by $R^{[+]} \triangleq (\{[x_0, \neg x_0,]x_1\} \vdash \neg x_1)$ is $\neg$-paraconsistent, in which case proper$|\neg$-paraconsistent extensions of C are exactly extensions$|$sublogics of $C^{R^{+|}} \subseteq | = C_{\frac{1}{2}}[(\varnothing) = C(\varnothing)]$, while C is not pre-maximally $\neg$-paraconsistent iff $C_{[\frac{1}{2}]}$ has no theorem iff C is not weakly disjunctive iff $2 \in \mathbf{S}\mathfrak{A}$ iff $C^{[R^+]}$ "is $\neg$-subclassical"/"has a consistent non-$\neg$-paraconsistent extension" iff $C_{\frac{1}{2}|}$ is not maximally$|$pre-maximally consistent iff $C_{\frac{1}{2}} \neq C^{R^+}$ iff $C_{\frac{1}{2}}$ is not the only proper [$\neg$-para]consistent extension of C, in which case proper $\neg$-paraconsistent both extensions/sublogics of $C_{/\frac{1}{2}}$ are exactly $|$"$\neg$-subclassical $\neg$-paraconsistent" extensions of C^{R^+} "with models $\mathcal{A} \upharpoonright 2$ and $\mathcal{A}_{\frac{1}{2}}$"$|$, whereas C is almost pre-maximally $\neg$-paraconsistent iff $C^{R^+}\|$there is a unique proper "both $\neg$-paraconsistent/-subclassical extension"$|$"$\neg$-paraconsistent both extension/sublogic" of $C_{|/\frac{1}{2}}$ iff proper $\neg$-paraconsistent extensions of C are exactly $C^{R^{[+]}}$ iff C^{R^+} is defined by $\{(\mathcal{A} \upharpoonright 2), \mathcal{A}_{\frac{1}{2}}\}$ iff $(\Delta_A \cup (\{\frac{1}{2}\} \times 2)) \in \mathbf{S}\mathfrak{A}^2$ and there is no secondary binary connective β of L such that $\forall a \in D, \forall b \in (2 \cdot a) : \beta^{\mathfrak{A}}(a, b) = (1 - (a \cdot (1 - b)))$, and so C is [/pre-]maximally $\neg$-paraconsistent, whenever it is weakly conjunctive/"disjunctive (i.e., has a theorem) and [not] $\wr$-subclassical".*

▸ ALEXEJ PYNKO* AND IRA SIRKO, *Extensions of paraconsistent three-valued chain logics.*
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Given any propositional language L (viz., a set of primary connectives, treated as operation symbols, when dealing with L-algebras), an *L-rule* is any expression of the form $\mathcal{R} = ([\Gamma \vdash] \varphi)$, where $[\Gamma \subseteq] \mathrm{Fm}_L \ni \varphi$, whereas Fm_L is the set of L-formulas with variables in $V = \{x_i\}_{i \in \omega}$ — viz. the carrier of the absolutely-free L-algebra $\mathfrak{Fm}_L$ freely-generated by V, natural numbers, including 0, being treated as sets of lesser ones, the set of all them being denoted by ω, while $\neg | \wedge / \vee \supset$ is a $(1|2)$-ary prefix$|$infix connective of L (possibly, a secondary one — viz., an L-formula with variables in $\{x_j\}_{j \in (1|2)}$). Then, an *L-logic* C (viz., a *structural* closure operator over Fm_L — i.e., with $\mathrm{img} C$ closed under inverse endomorphisms of $\mathfrak{Fm}_L$) is said to "*satisfy* $\mathcal{R}$"$|$"be *[$\neg$-para]consistent*", if $(\varphi | x_1) \in | \notin C(\varnothing[\cup(\Gamma|\{x_0, \neg x_0\})])$, an *extension of* C (viz., an L-logic C' with $(\mathrm{img} C') \subseteq (\mathrm{img} C)$) being said to be *proper*/"*relatively axiomatized by* $\mathcal{R}$", if it is "distinct from C"/"the least extension of C satisfying $\mathcal{R}$". Likewise, any *L-matrix* (viz., a pair $\mathcal{A} = \langle \mathfrak{A}, D \rangle$, constituted by its *underlying* L-algebra $\mathfrak{A}$ with carrier A, consisting of its *values*, and the set $D \subseteq A$ of its *distinguished* values) *defines* its logic

$\mathrm{Cn}_{\mathcal{A}}$ such that $\{h^{-1}[D] \mid h \in \hom(\mathfrak{Fm}_L, \mathfrak{A})\}$ is a closure basis of $\mathrm{img}\mathrm{Cn}_{\mathcal{A}}$, as well as said to be $\neg$-*classical*$|$ $\supset$-*implicative*, if "it has exactly $2[-1]$ [distinguished] values, while $\neg^{\mathfrak{A}}$ permutes its unique distinguished and non-distinguished values"$|\forall a, b \in A : ((a \in D) \Rightarrow (b \in D)) \Leftrightarrow ((a \supset^{\mathfrak{A}} b) \in D)$. Then, C is said to be $\neg$-*classical*$|$ $\supset$-*implicative*, if "it is defined by a $\neg$-classical L-matrix, in which case it is consistent but not $\neg$-paraconsistent"$|\forall \Delta \subseteq \mathrm{Fm}_L, \forall \phi, \psi \in \mathrm{Fm}_L : (\psi \in C(\Delta \cup \{\phi\})) \Leftrightarrow ((\phi \supset \psi) \in C(\Delta))$, L-logics with $\neg$-classical extensions being referred to as $\neg$-*subclassical.*

THEOREM 1. *Let $\mathfrak{A}$ be an L-algebra with carrier $A \triangleq (2 \cup \{\frac{1}{2}\})$, $a \triangleq \neg^{\mathfrak{A}} \frac{1}{2}$, $b \in \{a, 1\}$, $D \subseteq (A \setminus 1)$ and $\mathcal{A} \triangleq \langle \mathfrak{A}, D \rangle$. Suppose $1 \in D$, while [the least subalgebra of] $\langle A, \wedge^{\mathfrak{A}}, \vee^{\mathfrak{A}}, 0, b[+(1-b), \neg^{\mathfrak{A}}] \rangle$ is a [complemented] bounded lattice, whereas $\mathrm{Cn}_{\mathcal{A}}$ is both $\neg$-paraconsistent (i.e., $\{\frac{1}{2}, a\} \subseteq D$) and $\{not\}$ non-$\neg$-subclassical (i.e., the subalgebra of $\mathfrak{A}$ generated by 2 does $\{not\}$ contain $\frac{1}{2}$) $\langle$as well as $\supset$-implicative (i.e., $\mathcal{A}$ is so)$\rangle$. Then, $\mathrm{Cn}_{\mathcal{A}}$ has no consistent proper extension $\{$other than $\mathrm{Cn}_{(\mathcal{A}\restriction 2)/(\mathcal{A} \times (\mathcal{A}\restriction 2))}$ relatively axiomatized by $(((\{x_0 \lceil \vee x_1 \rceil, \neg x_0 \vee x_1\} | \{x_0 \vee x_1, \neg x_0\}) / \{x_0, \neg x_0\}) \vdash ([\neg\neg x_0 \vee] x_1) / ([\lfloor x_1 \vee \rfloor \neg\neg]$ $x_{1[-1]}))\langle \| \neg x_0 \supset ||| \vdash (x_0 \supset ([\neg\neg] x_{1[-1]}))/\rangle$ [unless $a = \frac{1}{2}$], in which case the former is a $\neg$-classical proper extension of the latter, and so the latter is not $\neg$-classical, while $\mathrm{Cn}_{\mathcal{A}}(\varnothing) =$
$mathrmCn_{\mathcal{A} \times (\mathcal{A}\restriction 2)}(\varnothing)$, whereas $\mathrm{Cn}_{\mathcal{A}}(\varnothing) \neq \mathrm{Cn}_{\mathcal{A}\restriction 2}(\varnothing)$ iff $\mathcal{A}/\!/\mathrm{Cn}_{\mathcal{A}}$ is implicative iff $\{\langle i, i \rangle \mid i \in 2\} \cup (\{\frac{1}{2}\} \times (\frac{1}{a}))$ does not form a subalgebra of $\mathfrak{A}^2$ iff either $b = \frac{1}{2}$ or $\mathfrak{A}$ has a (dual) discriminator$\}$.*

This covers arbitrary *three-valued* expansions of "the *logic of paradox LP*" / "Hałkowska-Zajac' logic HZ" (with $a = \frac{1}{2}$ and $b = (1/\frac{1}{2})$ / "as well as secondary binary connectives $\neg x_0(\vee|\wedge)\neg x_1$ for primary ones $\wedge|\vee$") and the $\neg$-paraconsistent counterpart of the implication-less fragment of Gödel's three-valued logic resulted from leaving non-distinguished 0 alone and taking dual pseudo-complement for pseudo-complement, in which case $(a|b) = 1$, thus subsuming results originally proved by PYNKO *ad hoc.*

▸ GEMMA ROBLES, *The logic E-Mingle and its Routley-Meyer semantics.*
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The logic R-Mingle (RM) is axiomatized when adding the "mingle axiom" (M: $A \to (A \to A)$) to Anderson and Belnap's logic of the relevant implication R. The logic E-Mingle (EM) is the result of adding the "restricted mingle axiom" (Mr: $(A \to B) \to [(A \to B) \to (A \to B)]$) to Anderson and Belnap's logic of entailment E (cf. [1]).

Contrary to what is the case with RM and its extensions, thoroughly investigated logics since the beginning of the "relevance enterprise" (cf. [1]), practically everything is ignored about EM. In particular, this logic lacks a semantics whatsoever. The aim of this paper is to remedy this deficiency by providing a Routley-Meyer semantics for EM, despite the fact that the creators of this semantics think that it is no possible to interpret Mr in it (cf. [2, §4.9]). EM is endowed with a Routley-Meyer semantics by giving it a Hilbert-style formulation in which Mr does not appear.

[1] A. R. ANDERSON, N. D. BELNAP, ***Entailment. The Logic of Relevance and Necessity***, vol. I, Princeton University Press, 1975.

[2] R. ROUTLEY, R. K. MEYER, V. PLUMWOOD, R. T. BRADY, ***Relevant Logics and their Rivals***, vol. 1, Ridgeview Publishing Co., Atascadero, CA. 1982.

▸ TAPIO SAARINEN, *The categoricity of complete theories in second-order logic.*
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A complete theory is said to be categorical, if it has a unique model up to isomorphism. Due to the upwards and downwards Löwenheim-Skolem theorems, complete first-order theories with infinite models are never categorical, as they have models in all infinite cardinalities. In contrast, the second-order versions of many familiar theories (such as second-order Peano Arithmetic, or the second-order theory of the real numbers as a complete ordered field) are categorical. One therefore wonders about the extent of this phenomenon: given an arbitrary complete second-order theory, is it categorical? As non-categorical complete second-order theories exist by a cardinality argument, it is reasonable to require that the theory is tractable in some sense (such as finitely axiomatizable, recursively axiomatizable, or that it has a model of size κ for some particular cardinal κ). It turns out that for many classes of theories, the answer is independent of ZFC.

In this talk we present some new results in this area.

► GÁBOR SÁGI, *Automorphism invariant measures on some structures and on their automorphism groups.*
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Let $\mathcal{A}$ be a countable $\aleph_0$-homogeneous structure. Our primary motivation is to study different amenability properties of (subgroups of) the automorphism group $Aut(\mathcal{A})$ of $\mathcal{A}$. The secondary motivation is to study the existence of weakly generic tuples of automorphisms of $\mathcal{A}$.

Among others, we present sufficient conditions implying the existence of automorphism invariant probability measures on certain subsets of A and $Aut(\mathcal{A})$. We also present sufficient conditions implying that the theory of $\mathcal{A}$ is amenable. More concretely, our main results are as follows.

THEOREM 1. *If the set of locally finite automorphisms of $\mathcal{A}$ is dense (in particular, if $\mathcal{A}$ has weakly generic tuples of automorphisms of arbitrary finite length), then there exists a finitely additive probability measure μ on the subsets of $\mathcal{A}$ definable with parameters such that μ is invariant under $Aut(\mathcal{A})$.*

THEOREM 2. *If $\mathcal{A}$ is saturated and the set of its locally finite automorphisms is dense (in particular, if $\mathcal{A}$ is saturated and has weak generics), then the theory of $\mathcal{A}$ is amenable.*

► SAM SANDERS, *Two computational clusters in ordinary mathematics.*
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I provide an overview of recent joint work with Dag Normann on the computability theory of ordinary mathematics ([2, 3]), as follows.

Given a finite set, perhaps the most basic questions are *how many* elements it has, and *which ones*? We study this question in Kleene's *higher-order computability theory*, based on his computation schemes S1-S9 ([1]). In particular, a central object of study is the higher-order functional Ω which on input a finite set of real numbers, list the elements as a finite sequence.

Perhaps surprisingly, the 'finiteness' functional Ω give rise to a *huge and robust* class of computationally equivalent operations, called the Ω-cluster. For instance, many basic operations on functions of *bounded variation* (BV) are part of the Ω-cluster, including those stemming from the well-known *Jordan decomposition theorem*. In addition, we identify a second cluster of computationally equivalent objects, called the Ω_1-cluster, based on the functional Ω_1, the restriction of Ω to singletons. We also show that both clusters include basic operations on *regulated* and *Sobolev space* functions, respectively a well-known super- and sub-class of the class of BV-functions.

Our objects of study are fundamentally *partial* in nature, and we formulate an elegant and equivalent λ-calculus formulation of S1-S9 to accommodate partial objects. The advantages of this approach are three-fold: proofs are more transparent in our λ-calculus approach, all (previously hand-waved) technical details can be settled easily, and we can show that Ω_1 and Ω are not computationally equivalent to any *total* functional.

[1] Longley, John and Normann, Dag, ***Higher-order Computability***, Theory and Applications of Computability, Springer, 2015.

[2] Normann, Dag and Sanders, Sam, *Betwixt Turing and Kleene*, ***Lecture Notes in Computer Science*** (Logical Foundations of Computer Science 2022), (Sergei Artemov and Anil Nerode, editors), vol. 13137, Springer, 2022, pp. 236–253.

[3] ———, *On the computational properties of basic mathematical notions*, submitted, arXiv:2203.05250.

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Forcing is ubiquitous in set theory but a lot of its general theory depends on the Axiom of Choice (AC). Nevertheless, forcing still serves as the main technique for extending models of ZF, with or without AC. We will present results about forcing when AC fails, particularly in relation to Dependent Choice (DC) and its preservation. This work is joint with A. Karagila.

► PHILIPP SCHLICHT, *Countable ranks at the first and second projective levels.*
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Transfinite derivations and computations induce rank functions on sets of reals. The complexity of these ranks typically lies at the first or second level of the projective hierarchy or in between them. We study arbitrary ranks of countable length at these levels. Using robust ordinals, a variant of stable ordinals from proof theory, we calculate the suprema of lengths of countable Π^1_1 ranks, Σ^1_2 ranks, Π^1_1 prewellorders and Σ^1_2 wellfounded relations, among others. They all equal the first Σ_2-robust ordinal τ, or equivalently, Kechris' ordinal γ^1_2. Furthermore, we obtain results towards a characterisation of those Σ^1_2 sets that admit countable ranks.

This is a joint project with Merlin Carl and Philip Welch.

► D. GIHANEE SENADHEERA, *Effective Concept Classes of PAC and PACi Incomparable Degrees and Jump Structure.*
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The Probably Approximately Correct (PAC) learning model is a machine learning model introduced by Leslie Valiant in 1984. Similar to Turing reducibility there is a reducibility to this learning model as well. The PACi means a less restricted version of PAC reducibility. Here i refers to the independence of the size and the computation time of the PAC reducibility. The ordering of concept classes under PAC reducibility is nonlinear, even when restricted to particular concrete examples. We recursively construct two c.e. effective concept classes of incomparable PACi degrees to show that there exist incomparable PACi degrees. Similarly, we can construct for PAC degrees which is analogous to incomparable Turing degrees. The priority construction method is used to construct the two concept classes, which was used by Friedburg and Muchnik in their proof of incomparable Turing degrees. It was necessary to deal with the size of an effective concept class thus we propose to compute the size of the effective concept class using Kolmogorov complexity. Furthermore, we explore the jump structure of effective concept classes similar to the Turing jump and progress toward embedding 1degrees.

[1] Wesley Calvert, *PAC Learning, VC Dimensions, and The Arithmetic Hierarchy*, ***Archive for Mathematical Logic***, vol. 54, no.7-8, pp. 871–883.

[2] Wesley Calvert, ***Mathematical Logic and Probability***, preprint.

[3] M.J. Kearns and U.V. Vazirani, ***An Introduction to Computational Learning Theory***, MIT Press, 1994.

[4] Ming Li and Paul Vitányi, ***An Introduction to Kolmogorov Complexity and Its Applications***, Springer, Switzerland, 2019.

[5] Robert Soare, ***Recursively Enumerable Sets and Degrees***, Springer-Verlag, New York, 1987.

▸ ANDREI SIPOŞ, *On extracting variable Herbrand disjunctions*.

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In 2005, Gerhardy and Kohlenbach [1] gave a new proof of the classical Herbrand theorem, by using the Shoenfield variant [3] of Gödel's *Dialectica* interpretation. Such proof interpretations usually serve as a loose analogue to Herbrand's theorem for systems which include arithmetical axioms; they play a central role in the research program of *proof mining*, given maturity by the school of Kohlenbach [2], where they are applied to ordinary mathematical proofs in order to uncover new information.

Even though proof interpretations usually produce terms expressible in sophisticated systems, it has been observed that sometimes the extracted terms may take the form of a classical Herbrand disjunction but of variable length. What we do here is to logically elucidate this empirical fact, by extending the proof of Gerhardy and Kohlenbach to theories which are on the level of first-order arithmetic, dealing with the corresponding recursors (used to interpret induction) through Tait's infinite terms [5].

The results presented in this talk may be found in [4].

[1] P. Gerhardy, U. Kohlenbach, *Extracting Herbrand disjunctions by functional interpretation*, ***Archive for Mathematical Logic***, vol. 44 (2005), no. 5, pp. 633–644.

[2] U. Kohlenbach, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[3] J. Shoenfield, ***Mathematical Logic***, Addison-Wesley Series in Logic, Addison-Wesley Publishing Co., 1967.

[4] A. Sipoş, *On extracting variable Herbrand disjunctions*, arXiv:2111.12133 [math.LO], 2021. To appear in: ***Studia Logica***.

[5] W. W. Tait, *Infinitely long terms of transfinite type*, ***Formal Systems and***

Recursive Functions (J. N. Crossley and M. A. E. Dummett, editors), Elsevier, Amsterdam, 1965, pp. 176–185.

▸ IOANNIS SOULDATOS, *Characterizing Cardinals by $\mathcal{L}_{\omega_1,\omega}$-sentences in an Absolute Way.*
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In [1], Hjorth proved that for every countable ordinal α, there exists a complete $\mathcal{L}_{\omega_1,\omega}$-sentence ϕ_α that has models of all cardinalities less than or equal to $\aleph_\alpha$, but no models of cardinality $\aleph_{\alpha+1}$. Unfortunately, his solution yields not one $\mathcal{L}_{\omega_1,\omega}$-sentence ϕ_α, but a set of $\mathcal{L}_{\omega_1,\omega}$-sentences, one of which is guaranteed to work.

The following is new: It is independent of the axioms of ZFC which of the Hjorth sentences works. More specifically, we isolate a diagonalization principle for functions from ω_1 to ω_1 which is a consequence of the *Bounded Proper Forcing Axiom* (BPFA) and then we use this principle to prove that Hjorth's solution to characterizing $\aleph_2$ in models of BPFA is different than in models of CH.

This raises the question whether Hjorth's result can be proved in an *absolute way* and what exactly this means, which we will discuss at the end of the talk.

This is joint work with Philipp Lücke.

[1] Greg Hjorth, *Knight's model, its automorphism group, and characterizing the uncountable cardinals*, ***Journal of Mathematical Logic***, vol. 2 (2002), no. 1, pp. 113–144.

[2] Philipp Lücke and Ioannis Souldatos, *A lower bound for the Hanf number for joint embedding*, preprint, arXiv:2109.07310

▸ SEBASTIAN G.W. SPEITEL, *Arithmetic via Carnap-categoricity.*
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The existence of non-standard models of first-order Peano-arithmetic (PA) has long been taken to undermine the claim of the mathematical realist that determinate reference to the natural number structure is possible in a non-mysterious, naturalistically acceptable way. The use of logics stronger that FOL to achieve a categorical theory of arithmetic and resolve this referential indeterminacy has been criticised as merely pushing the issue 'one level up' into the meta-theory of these logics. This, the model-theoretic sceptic claims, is due to the fact that the resources needed to formulate these logics are just as much in need of justification as reference to the natural number structure itself.

In [1] we outlined and defended a novel criterion of logicality based on the idea that logical notions must be *formal* (invariant under isomorphisms) as well as *categorical* (uniquely determinable by inference). A notion satisfying this criterion was termed *Carnap-categorical*. In this talk, I want to show that our criterion offers an attractive and well-motivated answer to the sceptical challenge advanced against the mathematical realist. The reply is based on the Carnap-categoricity of the generalised quantifier "there are infinitely many" ($\mathcal{Q}_0$). It is this property of $\mathcal{Q}_0$ which allows us to successfully mitigate the objection that the indeterminacy affecting reference to the natural number structure simply re-arises at the level of the meta-theory of the logic used to provide a categorical axiomatization of that structure.

I compare this approach with other attempts to justify the move to stronger logics found in the literature and argue that the proposal based on Carnap-categoricity is more robust and thus preferable. I conclude by reflecting on the scope of the response to the sceptical challenge and the remaining sources of indeterminacy.

[1] D. Bonnay, S.G.W. Speitel, *The Ways of Logicality: Invariance and Categoricity*, ***The Semantic Conception of Logic. Essays on Consequence, Invariance, and Meaning*** (G. Sagi and J. Woods, editors), Cambridge University Press, 2021, pp. 55–79.

▸ WILL STAFFORD, *Compositional proof-theoretic semantics for natural language.*
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Francez and co-authors [1, 2, 3] attempt to offer a proof-theoretic semantics for natural language. They take the meaning of sentences to be functions from sets of assumptions (or hypothesis or premises) to canonical proofs of the sentence. A proof for Francez is canonical if it ends in an introduction rule. Using this it has been shown for a fragment of English that one gets the expected behaviour for the grammatical type of the words considered. Francez proposal distinguishes between the meaning of words given in terms of proof rules and the meanings of sentences given in terms of canonical proofs. But to do this a third element must be used which is the set of all proofs of a sentences and called the "sentence contribution". This makes the semantics not compositional. It will be argued that the lack of compositionality here is a problem, as we want compositionality for learnability or effectiveness, it will be argued that Francez suggestion to use sentence contributions is unacceptable. I propose an alternative sentence meaning given by the set of proofs in normal form. A proof in normal form is one where all possible elimination rules are applied before introduction rules are. It then follows that we can give compositional meanings without using the sentence contributions. This is a small tweek but it allows for a compositional presentation. The presentation will argue that this definition of propositions is preferable to Francez's original presentation.

[1] Francez, N and Dyckhoff, R, *Proof-theoretic semantics for a natural language fragment*, ***Linguistics and Philosophy***, vol. 33 (2010), no. 6, pp. 447–477.
[2] Francez, N and Ben-Avi, G, *Proof-theoretic reconstruction of generalized quantifiers*, ***Journal of Semantics***, vol. 32 (2015), no. 3, pp. 313–371.
[3] Nissim Francez, ***Proof-Theoretic Semantics***, College Publications, 2015.

▸ DOROTTYA SZIRÁKI, *Applications of the open dihypergraph dichotomy for generalized Baire spaces.*
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The open graph dichotomy for a given subset X of the Baire space ${}^{\omega}\omega$ states that any open graph on X either contains a large complete subgraph or admits a countable coloring. It is a definable version of the open coloring axiom for X and it generalizes the perfect set property. Miller, Carroy and Soukup showed that several well-known results regarding the second level of the Borel hierarchy follow from an infinite dimensional generalization of the open graph dichotomy.

We show that several of these applications, including the Hurewicz dichotomy and the Kechris-Louveau-Woodin dichotomy, can be lifted to the generalized Baire space ${}^{\kappa}\kappa$, where κ is an uncountable cardinal with $\kappa^{<\kappa} = \kappa$. We also obtain new applications, such as the determinacy of Väänänen's perfect set game for all subsets of ${}^{\kappa}\kappa$ and an asymmetric version of the Baire property. These results extend previous work of Lücke, Motto Ros, Schlicht, Väänänen and the author. This is joint work with Philipp Schlicht.

▸ TINKO TINCHEV, *Decidability of modal definability problem on the class of quasilinear frames.*
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Let $\mathcal{K}$ be the class of all quasilinear Kripke frames, i.e. the accesibility relation is reflexive, transitive and total (linear). Denote by $\mathcal{K}^{\text{fin}}$ and by $\mathcal{K}^{\leq\omega}$ the classes of frames from $\mathcal{K}^{\text{fin}}$ having finetely many, resp. at most countably many, clusters. Our goal is to study the modal definability problem on these three classes. Remind that a sentence A from the first-order language with equality and one binary predicate symbol is modally definable with respect to some class of frames if there is a modal formula φ from the classical propositional modal language such that A and φ are valid in the same frames from the class. Modal definability problem ask whether there exists an algorithm that recognizes modally definable sentences.

In this talk we make a heavy use of decidability of Rabins's $S2S$ theory to prove the following.

Theorem 1. *The modal definability problem is decidable with respect to the classes* $\mathcal{K}^{\text{fin}}$ *and* $\mathcal{K}^{\leq\omega}$.

Theorem 2. *For any sentence A, A is modally definable with respect to $\mathcal{K}$ if and only if A is modally definable with respect to $\mathcal{K}^{\leq\omega}$. Therefore, the modal definability problem with respect to $\mathcal{K}$ is decidable.*

Theorem 3. *1. There is an algorithm which for any sentence A gives a modal definition of A on $\mathcal{K}^{\text{fin}}$, if such modal formula exists.*
2. There is an algorithm which for any sentence A gives a modal definition of A on $\mathcal{K}$, if such modal formula exists.

▸ AGATA TOMCZYK, *Sequent Calculus for non-Fregean Boolean theory* WB.
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The aim of the talk is to present Sequent Calculus for WB—a Boolean extension of the weakest non-Fregean logic SCI (*Sentential Calculus with Identity*) proposed by Roman Suszko [2]. In WB we consider identity connective '$\equiv$' based on one introduced in SCI. However, WB consists of more tautological identities than SCI, where the only tautological identity was of the form $\phi \equiv \phi$. In case of WB, $\phi \equiv \chi$ is a tautology if and only if $\phi \leftrightarrow \chi$ is a tautology of Classical Propositional Calculus. To formalize this notion we introduce proof system $\mathsf{G3}_{\mathsf{WB}}$ (based on $\ell\mathbf{G}3_{\mathsf{SCI}}$ found in [1]) in which each sequent is labelled with marker allowing (or disabling) the use of certain identity-dedicated rules. We will discuss correctness and invertibility of the proposed rule set and identify issues regarding the cut elimination procedure. We will also discuss ideas concerning sequent calculus for WT, a topological Boolean algebra of situations [2].

[1] Szymon Chlebowski, *Sequent Calculi for SCI*, ***Studia Logica***, vol. 106 (2018), no. 3, pp. 541–563.

[2] Roman Suszko, *Abolition of the Fregean Axiom*, ***Lecture Notes in Mathematics***, vol. 453 (1975), pp. 169–239.

▸ PATRICK UFTRING, *Weak and strong versions of effective transfinite recursion.*
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Working in the context of reverse mathematics, we give a fine-grained characterization result on the strength of two possible definitions for Effective Transfinite Recursion used in literature. Moreover, we show that Π^0_2-induction along a well-order X is equivalent to the statement that the exponentiation of any well-order to the power of X is well-founded.

▶ HAFIZ ULLAH, *Weakly Menger Ditopological Texture Spaces.*
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Weakly Menger spaces were introduced by B. A. Pansera in topological spaces. We extend this idea to define weakly-s-Menger ditopological texture spaces. Also we study the interrelation between s-Menger, Weakly-s-Menger and almost-s-Menger ditopological texture spaces. We have characterized some preservations of these notion under direlation, difunction and various type of mappings.

[1] L. M. Brown and M. Diker, *Ditopological texture spaces and intuitionistic sets*, ***Fuzzy Sets and Systems***, vol. 98 (1998), pp. 217–224.

[2] L. M. Brown, R. Ertürk and ¸S. Dost, *Ditopological texture spaces and fuzzy topology, II. Topological Considerations*, ***Fuzzy Sets and Systems***, vol. 147 (2004), no. 2, pp. 201–231.

[3] Lj. D. R. Kočinac, *Star-Menger and related spaces II*, ***Filomat***, vol. 13 (1999), pp. 129–140.

[4] P. Staynova, *Weaker forms of the Menger property*, ***Quaest. Math***, vol. 35 (2012), pp. 161–169.

▶ CHENG-SYUAN WAN, *Proof Theory of Skew Non-Commutative* MILL.
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Monoidal closed categories are models of non-commutative multiplicative intuitionistic linear logic (NMILL). Skew monoidal closed categories are weak variants of monoidal closed categories [2]. In the skew cases, three natural isomorphisms $\lambda : \mathsf{I} \otimes A \cong A$, $\rho : A \cong A \otimes \mathsf{I}$, and $\alpha : (A \otimes B) \otimes C \cong A \otimes (B \otimes C)$ are merely natural transformations with a specific orientation. In previous works by Uustalu et al. [3] [4], proof theoretical analysis on skew monoidal categories and skew closed categories are investigated. In particular, the sequent calculus systems modelled by skew monoidal and skew closed categories are respectively constructed. Moreover, proof theoretical semantics of each system is provided according to Jean-Marc Andreoli's focusing technique [1].

Following the results above, a question arises: is it possible to construct a sequent calculus system naturally modelled by skew monoidal closed categories? We answer the question positively by constructing a cut-free system $\texttt{NMILL}^s$, a skew version of NMILL. Furthermore, we study the proof theoretical semantics of $\texttt{NMILL}^s$. The inspiration also originates from focusing, but we peculiarly employ tag annotations to keep tracking new formulae occurring in antecedent and reducing non-deterministic choices in bottom-up proof search. Focusing solves the coherence problem of skew monoidal closed categories by providing a decision procedure to determine equality of maps in the free skew monoidal closed category.

This is joint work with Tarmo Uustalu (Reykjavik University) and Niccolò Veltri (Tallinn University of Technology).

[1] Jean-Marc Andreoli, *Logic Programming with Focusing Proofs in Linear Logic.*, ***Journal of Logic and Computation***, vol.2(3), pp. 297–347.

[2] Ross Street, *Skew-Closed Categories.*, ***Journal of Pure and Applied Algebra***, vol.217(6), pp. 973–988.

[3] Tarmo Uustalu, Niccolò Veltri, and Noam Zeilberger, *Deductive Systems and Coherence for Skew Prounital Closed Categories.*, ***Eletronic Proceedings in Theoretical Computer Science***, vol.332, pp. 35–53.

[4] ———, *The Sequent Calculus of Skew Monoidal Categories.*, ***Joachim Lambek: The Interplay of Mathematics, Logic, and Linguistics***, pp. 377–406.

▸ ANDREAS WEIERMANN, *The phase transition for Harvey Friedman's monotone Bolzano Weierstrass principle.*
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Let f be a weakly monotone and unbounded number-theoretic function. Harvey Friedman's monotone Bolzano Weierstrass principle with respect to f is the following assertion (MBW_f). $\forall K \geq 3\exists M\forall x_1,\ldots,x_M \in [0,1](x_1 < \ldots < x_M \to \exists k_1,\ldots,k_K(k_1 < \ldots < k_K \to \forall L \leq K-2 \mid x_{k_{l+1}} - x_{k_{l+2}} \mid < \frac{1}{f(k_l)}))$. Friedman has shown that MBW_f is true (by an application of the compactness of the Hilbert cube). Morever Friedman has shown that for $f(x) = 2^x$ the principle MBW_f is provable from $I\Sigma_1 + \forall x\exists y A(x,0) = y$ where A is the Ackermann function. In our talk we will approximate the phase transition for MBW_f and for this we will apply classical results by Abel (and its refinement by Elstrodt and Fischer) on the convergence of logarithmic series. In particular we will show that $I\Sigma_1 \vdash MBW_f$ for $f(i) = i\cdot\log(i)\cdot\ldots\cdot\log_{\log^*(i)}(i)$ where $\log^*(i)$ is the functional inverse of the tower function.

▸ AGNIESZKA WIDZ, *What are magic sets?.*
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Given a family of real functions F we say that a set $M \subset \mathbb{R}$ is magic for F if for all $f, g \in F$ we have $f[M] \subset g[M] \Longrightarrow f = g$. This notion was introduced by Diamond, Pomerance and Rubel in 1981 [1] and investigated by many mathematicians, including S. Shelah, K. Ciesielski and M. Burke. Recently some results about magic sets were proved by Halbeisen, Lischka and Schumacher [2]. Inspired by their work I constructed two families of magic sets one of them being almost disjoint and the other one being independent. During my talk I will sketch the background, present the construction of one of those families, and show some general properties of magic sets.

▸ KENTARO YAMAMOTO, *The strong small index property for the Fraïssé limit of finite Heyting algebras.*
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The small index property of countable ultrahomogeneous lattice-based structures has been established in several cases such as the countable Boolean algebras (Truss) and the universal homogeneous distributive lattice (Droste and Macpherson). In this work, the strong small index property of the Fraïssé limit L of finite Heyting algebras is proved. This is the prime model of the model-completion of the theory of Heyting algebras, which is not ω-categorical unlike in the aforementioned existing literature. The method used is an adaptation of the ad-hoc argument used by Truss for the countable atomless Boolean algebra and is based on the author's previous result (under review) showing the simplicity of the automorphism group of L.

Abstract of talk presented by title

▶ PAVEL ARAZIM, *The limits of expressing logic.*
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In his Tractatus, Wittgenstein dedicates some of the most fascinating, yet also most enigmatic passages to the sphere of the mystical. One of the characteristics of this sphere is supposed to be its ineffability. Any attempts to describe it force us to maim the expressive powers of the language we use. Surpsingly enough, Wittgenstein treats logic in a very similar way in Tractatus. Logic, then, can only be shown, not expressed. Or, to be more precise, logic can only show itself. Besides being ineffable, the mystical is also supposed to be fundamental, in fact much more important than what lies outside it. Therefore, logic also deserves this honourable status, according to Wittgenstein. Nevertheless, logicians today purport to be making explicit all kinds of logical laws which hold in variegated areas, which causes the unprecedented plurality of logics. On the other hand, it is not clear what the import of all this intellectual work is. Is there a lesson to be learned from Wittgenstein for the contemporary philosophy of logic? In order to access this possible lesson, we have to pay attention not only to early Wittgenstein but also to his later development where the notion of game and language game became prominent. I will show that taking seriously Wittgenstein´s motivation - which originates in his discussions with Moritz Schlick and his conception of games - to treat our linguistic activities as games, which are partly playful and unserious, shows us the limits of formal logical systems. They are language games themselves but do not understand themselves properly which causes them to be unsatisfying and turns the plurality of logics into a curse rather than into a blessing, getting us close to the positions of logical nihilists rather than to those of logical pluralists.

[1] RUSSELL, G., *Logical nihilism: Could there be no logic?*, ***Philosophical Issues***, vol. 28 (2018), no. 1.

[2] SCHLICK, M., *Vom Sinne des Lebens*, ***Die Wiener Zeit: Aufsätze, Beiträge und Rezensionen 1926-1936*** (J. Friedl and H. Rutte, editors), Springer, 2008.

[3] WITTGENSTEIN, L., *Logisch-Philosophische Adhandlung*, ***Annalen der Naturphilosophische***, vol. XIV (1921), no. 3-4.

[4] WITTGENSTEIN, L., ***Philosophische Untersuchungen***, Blackwell, Oxford, 1953.

▶ JOACHIM MUELLER-THEYS, *The inhomogeneity of concepts.*
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We may think of $P, Q, \ldots \subseteq M$ as properties or concepts. For $a, b, \ldots \in M$, $P(a)$:iff $a \in P$. P is *total* :iff $P = M$, P *vacuous* :iff $-P$total,P *real* :iff P not vacuous. We naturally call P *extreme* :iff P is vacuous or total iff. P real implies P total. P *singular* :iff $|P| = 1$. We naturally call P *genuine* iff P is neither vacuous nor singular.

We have defined P-*similarity* $a \sim_P b$ by $P(a)$ & $P(b)$ and P-*equality* $a \equiv_P b$ by $P(a) \Leftrightarrow P(b)$. The basic connection is $\sim_P \subseteq \equiv_P$; the converse is not true in general.

We say that Q *differentiates* P :iff $P(a)$, $P(b)$, but $a \not\equiv_Q b$ for some a, b. For example, evil differentiates human, transuranic differentiates element. We call P *inhomogeneous* iff there exists Q such that Q differentiates P. Accordingly, human and element are inhomogeneous. We have found that, in general, *all genuine concepts are inhomogeneous*: Let P be genuine, whence $|P| \geq 2$, whereby $P(a)$, $P(b)$ for some $a \neq b$. Now let $Q := P \setminus \{b\}$, whence $Q(a)$, but non $Q(b)$, whereby $a \not\equiv_Q b$. Thus $\mathrm{Diff}_Q(P)$, whence

Inhom(P).

Inhom(P) may be seen as formalisation of sayings of the form "P *is not* P", like human is not human, element is not element. Phrases of the form "P *is* P", like "human is human", may be precisefied by $a \equiv_P b$ for all $a, b \in P$, which is a special case of *"all P are Q-equal"*: $\mathrm{AllEq}_Q(P)$ iff non $\mathrm{Diff}_Q(P)$. $Dicho_Q(P) := P \subseteq Q$ or P, Q disjoint. We had found and proven the *Dichotomy Theorem*: $\mathrm{AllEq}_Q(P)$ *iff* $\mathrm{Dicho}_Q(P)$.

Since $P \subseteq -Q$ iff $P \cap Q = \emptyset$, $Hom_Q(P) := P \subseteq Q$ or $P \subseteq -Q$ iff $\mathrm{Dicho}_Q(P)$, whence $\mathrm{AllEq}_Q(P)$ *iff* $\mathrm{Hom}_Q(P)$. Now as *corollaries*, $\mathrm{AllEq}_P(M)$ *iff* $\mathrm{Extr}(P)$, whereby human beings are equal only with respect to human, and Inhom(P) iff there is Q with $\mathrm{Inhom}_Q(P)$. Moreover, $\mathrm{Inhom}_Q(P)$ coincides with *heterogeneity* $Het_Q(P) := P \cap Q \neq \emptyset$ & $P \cap -Q \neq \emptyset$.

A sophisticated formal interpretation of "P *is* P" may now be $\forall Q$: $\mathrm{Hom}_Q(P) \Rightarrow \mathrm{AllEq}_Q(P)$ ("all P are equal with respect to all homogeneous Q"). It is curious that "P *is* P" and "P *is not* P if P is genuine" are tautologies both.

Joint work with Wilfried Buchholz. For related achievements and acknowledgments, see "Similarity and equality" (abstract 2021 North American Annual Meeting of the Association for Symbolic Logic, The Bulletin of Symbolic Logic 27 (2021), p. 329), "Mathematical theorems on equality and unequality" (abstract, Logic Colloquium 2021, by title), "Equivalence" (2022 ASL Annual Meeting).