

# A Formal Model for Direct-style Asynchronous Observables

Philipp Haller

KTH Royal Institute of Technology, Sweden

Heather Miller

EPFL, Switzerland

27th Nordic Workshop on Programming Theory (NWPT)  
Reykjavik University, Iceland, 21-23 October 2015



# Context: Asynchronous Programming

- Thread-based, blocking abstractions
  - Direct-style programming model (easy of use), good debugging support
  - Not efficient, not scalable
- Event-based, non-blocking abstractions
  - Efficient, scalable
  - Hard to use: inversion of control, “callback hell”
  - Debugging support lacking

# Background: Async Model

- A recent proposal for simplifying asynchronous programming
- Essence of the Async Model:
  1. A way to spawn an asynchronous computation (*async*), returning a (first-class) future
  2. A way to suspend an asynchronous computation (*await*) until a future is completed
- Result: a *direct-style API for non-blocking futures*
- Practical relevance: F#, C# 5.0, Scala 2.11

# Example

- Setting: Play Web Framework
- Task: Given two web service requests, when both are completed, return response that combines both results:

```
val futureDOY: Future[Response] =  
  WS.url("http://api.day-of-year/today").get  
val futureDaysLeft: Future[Response] =  
  WS.url("http://api.days-left/today").get
```

# Example

## Using Scala Async

```
val respFut = async {  
  val dayOfYear = await(futureDOY).body  
  val daysLeft = await(futureDaysLeft).body  
  Ok("" + dayOfYear + ": " + daysLeft + " days left!")  
}
```

# Example

## Using plain Scala futures

```
futureDOY.flatMap { doyResponse =>
  val dayOfYear = doyResponse.body
  futureDaysLeft.map { daysLeftResponse =>
    val daysLeft = daysLeftResponse.body
    Ok("" + dayOfYear + ": " + daysLeft + " days left!")
  }
}
```

## Using Scala Async

```
val respFut = async {
  val dayOfYear = await(futureDOY).body
  val daysLeft = await(futureDaysLeft).body
  Ok("" + dayOfYear + ": " + daysLeft + " days left!")
}
```

# Problem

- Async model only supports futures
- What about streams of asynchronous events?

# Asynchronous Streams

# Asynchronous Streams

- Asynchronous event streams and push notifications:  
a fundamental abstraction for web and mobile apps

# Asynchronous Streams

- Asynchronous event streams and push notifications: a fundamental abstraction for web and mobile apps
- Requirement: ***extreme scalability and efficiency***
  - Precludes future-per-event implementations
  - Examples: Netflix, Samsung SAMI, ...

# Asynchronous Streams

- Asynchronous event streams and push notifications: a fundamental abstraction for web and mobile apps
- Requirement: ***extreme scalability and efficiency***
  - Precludes future-per-event implementations
  - Examples: Netflix, Samsung SAMI, ...
- Popular programming model: Reactive Extensions
  - Based on the duality of iterators and observers

# Reactive Extensions: Essence

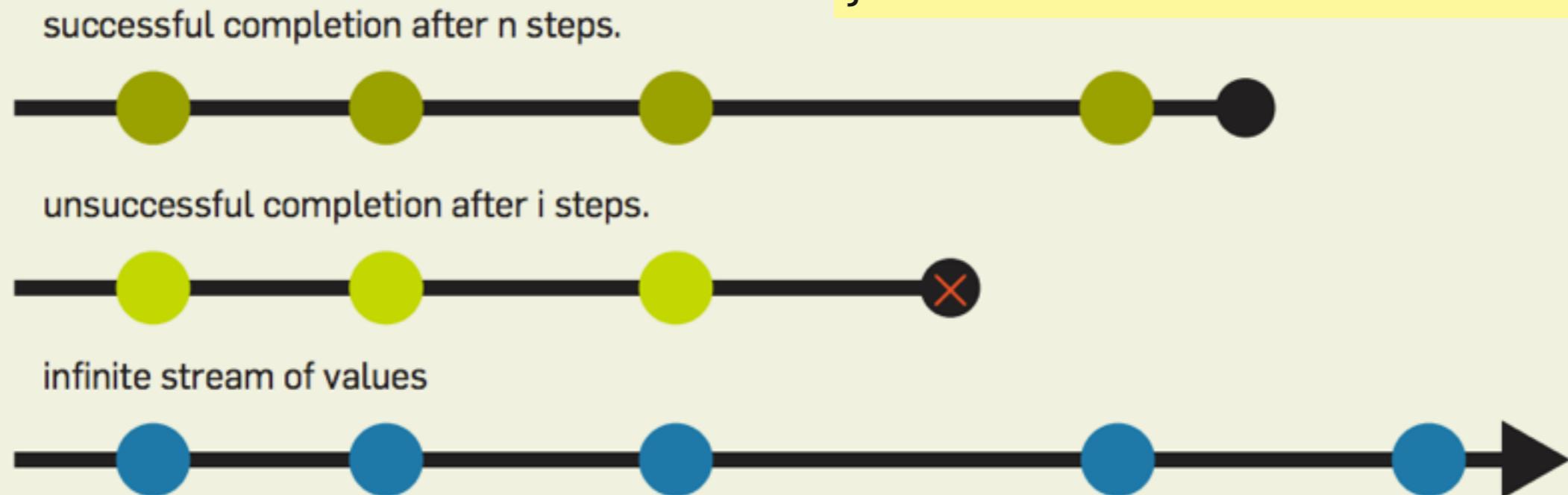
```
trait Observable[T] {  
  def subscribe(obs: Observer[T]): Closable  
}
```

```
trait Observer[T] {  
  def onNext(v: T): Unit  
  def onFailure(t: Throwable): Unit  
  def onDone(): Unit  
}
```

# Observer[T]: Interactions

```
trait Observer[T] {  
  def onNext(v: T): Unit  
  def onFailure(t: Throwable): Unit  
  def onDone(): Unit  
}
```

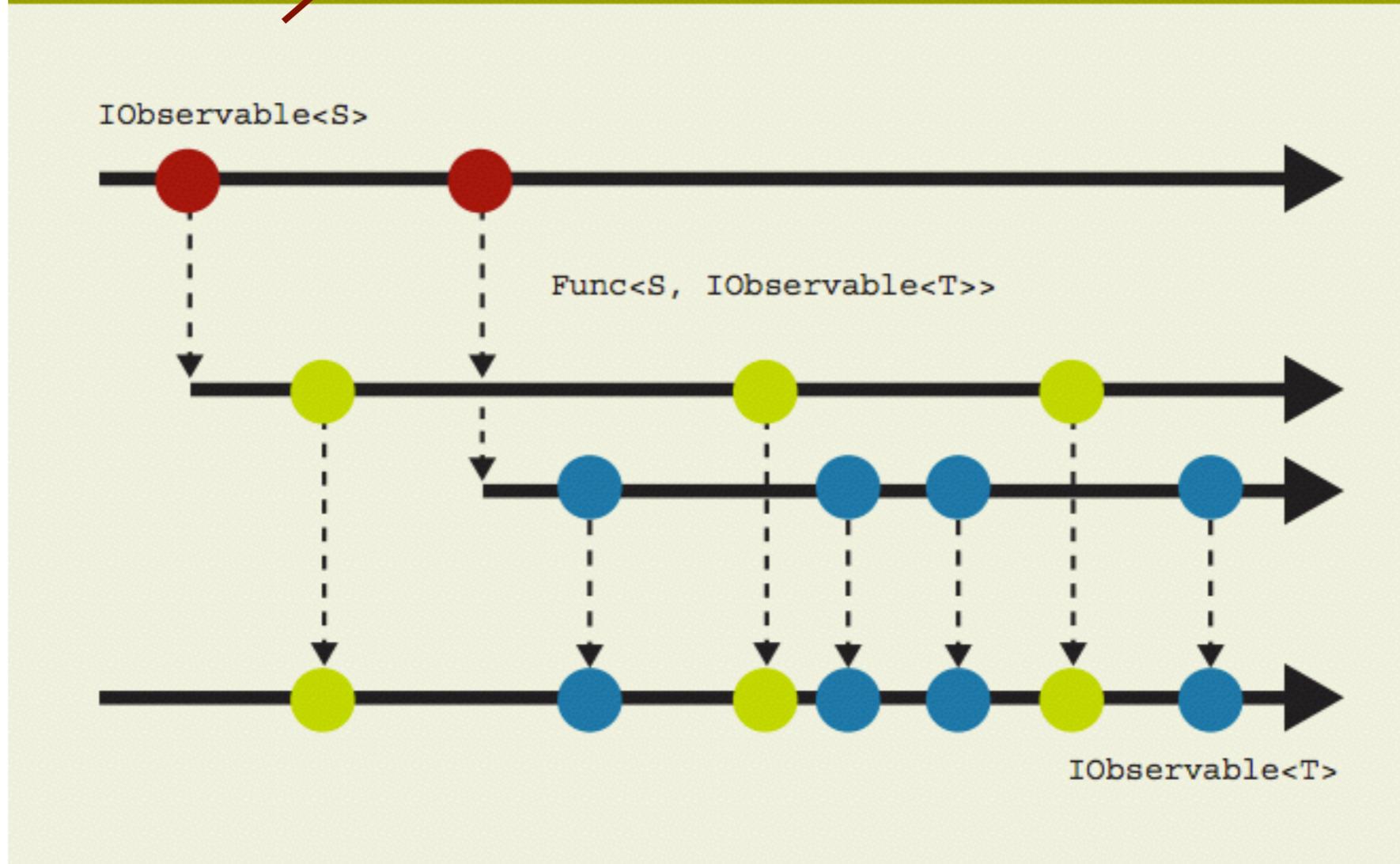
Figure 3. Possible sequences of interaction when using Observer[T].



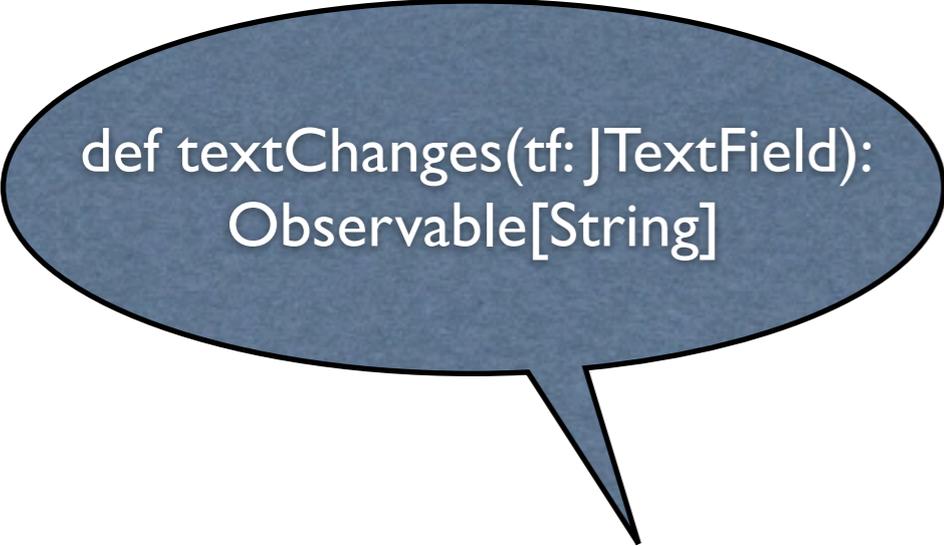
# The Real Power: Combinators

**flatMap**

Figure 7. The SelectMany operator.

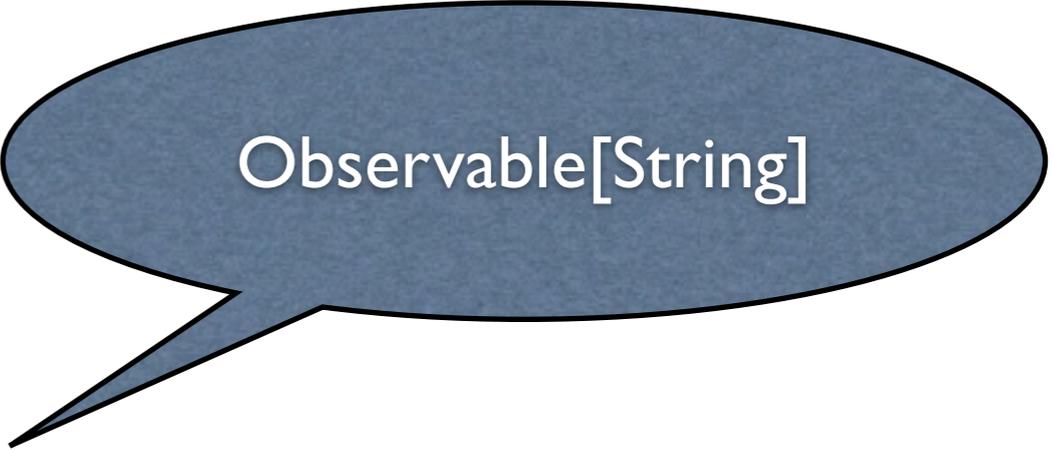


# Combinators: Example



```
def textChanges(tf: JTextField):  
  Observable[String]
```

```
textChanges(textField)  
  .flatMap(word => completions(word))  
  .subscribe(observeChanges(output))
```



```
Observable[String]
```

# Problem

- Programming with reactive streams suffers from an inversion of control
- Requires explicit programming in continuation-passing style (CPS)
- Writing stateful combinators is difficult

# RAY: Idea

- Unify Reactive Extensions and Async
- Introduce variant of `async { }` to create observables instead of futures: `rasync { }`
- Within `rasync { }`: enable *awaiting events of observables in direct-style*
- Create observables by yielding events from within `rasync { }`

# RAY: Primitives

- `rasync[T] { }` - create `Observable[T]`
- `awaitNextOrDone(obs)` - awaits and returns `Some(next event of obs)`, or else returns `None` if `obs` has terminated
- `yieldNext(evt)` - yields next event of current observable

# RAY: Simple Example

```
val forwarder = rasync[Int] {  
  var next: Option[Int] = awaitNextOrDone(stream)  
  while (next.nonEmpty) {  
    yieldNext(next)  
    next = awaitNextOrDone(stream)  
  }  
}
```

# Formalization

## Object-based calculus

|  |                    |
|--|--------------------|
| $p ::= \overline{cd} e$                                    | program            |
| $cd ::= \text{class } C \{ \overline{fd} \overline{md} \}$ | class declaration  |
| $fd ::= \text{var } f : \sigma$                            | field declaration  |
| $md ::= \text{def } m(\overline{x : \sigma}) : \tau = e$   | method declaration |
| $\sigma, \tau ::=$   | type               |
| $\gamma$   | value type         |
| $\rho$   | reference type     |
| $\gamma ::=$   | value type         |
| <b>Boolean</b>   | boolean            |
| <b>Int</b>   | integer            |
| $\rho ::=$   | reference type     |
| $C$  | class type         |
| <b>Observable</b> $[\sigma]$                               | observable type    |

# Expressions

| $e ::=$  | expressions         |
|--|---------------------|
| $\underline{b}$  | boolean             |
| $\underline{i}$  | integer             |
| $x$  | variable            |
| <code>null</code>  | null                |
| <code>if (x) {e} else {e'}</code>                                    | condition           |
| <code>while (x) {e}</code>   | while loop          |
| $x.f$  | selection           |
| $x.f = y$  | assignment          |
| $x.m(\bar{y})$   | invocation          |
| <code>new C(<math>\bar{y}</math>)</code>                             | instance creation   |
| <code>let x = e in e'</code>   | let binding         |
| <code>rasync [<math>\sigma</math>] (<math>\bar{y}</math>) {e}</code> | observable creation |
| <code>await(x)</code>  | await next event    |
| <code>yield(x)</code>  | yield event         |

# Expressions

| $e ::=$  | expressions         |
|--|---------------------|
| $\underline{b}$  | boolean             |
| $\underline{i}$  | integer             |
| $x$  | variable            |
| <code>null</code>  | null                |
| <code>if (x) {e} else {e'}</code>                                  | condition           |
| <code>while (x) {e}</code>   | while loop          |
| $x.f$  | selection           |
| $x.f = y$  | assignment          |
| $x.m(\bar{y})$   | invocation          |
| <code>new C(<math>\bar{y}</math>)</code>                           | instance creation   |
| <code>let x = e in e'</code>                                       | let binding         |
| <code>rasync[<math>\sigma</math>](<math>\bar{y}</math>) {e}</code> | observable creation |
| <code>await(x)</code>  | await next event    |
| <code>yield(x)</code>  | yield event         |

# Operational Semantics

- Small-step operational semantics
- Transitions for frames, frame stacks, and processes (sets of frame stacks)

$$\frac{H(L(y)) = \langle \rho, FM \rangle}{H, \langle L, \text{let } x = y.f \text{ in } e \rangle^l \longrightarrow H, \langle L[x \mapsto FM(f)], e \rangle^l} \quad (\text{E-FIELD})$$

$$\frac{\begin{array}{l} \text{fields}(C) = \bar{f} \quad o \notin \text{dom}(H) \\ H' = H[o \mapsto \langle C, \bar{f} \mapsto L(\bar{y}) \rangle] \end{array}}{H, \langle L, \text{let } x = \text{new } C(\bar{y}) \text{ in } e \rangle^l \longrightarrow H', \langle L[x \mapsto o], e \rangle^l} \quad (\text{E-NEW})$$

# Reducing Frame Stacks

$$\frac{\begin{array}{l} H(L(y)) = \langle \rho, FM \rangle \quad mbody(\rho, m) = (\bar{x}) \rightarrow e' \\ L' = [\bar{x} \mapsto L(\bar{z}), \mathbf{this} \mapsto L(y)] \end{array}}{H, \langle L, \mathbf{let } x = y.m(\bar{z}) \mathbf{ in } e \rangle^l \circ FS \rightarrow H, \langle L', e' \rangle^s \circ \langle L, e \rangle_x^l \circ FS} \quad (\text{E-METHOD})$$

$$\frac{}{H, \langle L, y \rangle^s \circ \langle L', e \rangle_x^l \circ FS \rightarrow H, \langle L'[x \mapsto L(y)], e \rangle^l \circ FS} \quad (\text{E-RETURN})$$

$$\frac{H, F \rightarrow H', F'}{H, F \circ FS \rightarrow H', F' \circ FS} \quad (\text{E-FRAME})$$

# Observables

- A special kind of object
- State of an observable: running or done

$$H(o) = \langle \text{Observable}[\sigma], \text{running}(\bar{F}, \bar{S}) \rangle$$

- Initial state:  $\text{running}(\epsilon, \epsilon)$
- Running state:  $\text{running}(\bar{F}, \bar{S})$
- Terminated state:  $\text{done}(\bar{S})$

# Waiters

$$H(o) = \langle \text{Observable}[\sigma], \text{running}(\bar{F}, \bar{S}) \rangle$$

# Waiters

$$H(o) = \langle \text{Observable}[\sigma], \text{running}(\bar{F}, \bar{S}) \rangle$$

- **Waiters:** asynchronous frames of suspended observables
- Example of a waiter:

$$F = \langle L, \text{let } x = \text{await}(y) \text{ in } t \rangle^{a(o, \bar{p})}$$

# Heap Evolution

Heap Evolution property formalizes ***permitted observable protocol state transitions***

**Definition 1** (Heap Evolution). Heap  $H$  evolves to  $H'$  wrt a set of observable ids  $B$ , written  $H \leq_B H'$  if

- (i)  $\forall o \in \text{dom}(H')$ . if  $o \notin \text{dom}(H)$  and  $H'(o) = \langle \psi, \text{running}(\bar{F}, \bar{S}) \rangle$  then  $\bar{F} = \bar{S} = \epsilon$ , and
- (ii)  $\forall o \in \text{dom}(H)$ .
  - if  $H(o) = \langle C, FM \rangle$  then  $H'(o) = \langle C, FM' \rangle$ ,
  - if  $H(o) = \langle \psi, \text{done}(\bar{S}) \rangle$  then  $H'(o) = \langle \psi, \text{done}(\bar{R} \uplus \{\langle o', q' \rangle\}) \rangle$  where  $\bar{S} = \bar{R} \uplus \{\langle o', q \rangle\}$ , and
  - if  $H(o) = \langle \psi, \text{running}(\bar{F}, \bar{S}) \rangle$  then  $H'(o) = \langle \psi, \text{running}(\bar{F}, \bar{S} \uplus \{\langle o, [] \rangle\}) \rangle$  or  $(H'(o) = \langle \psi, \text{running}(\epsilon, \bar{R}) \rangle$  and  $\text{dom}(\bar{S}) = \text{dom}(\bar{R})$ ) or  $(H'(o) = \langle \psi, \text{running}(\bar{F} \cup \bar{G}, \bar{S}) \rangle$ ,  $\text{obsIds}(\bar{F}) \# \text{obsIds}(\bar{G})$  and  $\text{obsIds}(\bar{G}) \subseteq B$ ) or  $H'(o) = \langle \psi, \text{done}(\bar{S}) \rangle$ .

# Example: Terminating an Observable

$$\begin{array}{l}
 H(o) = \langle \text{Observable}[\sigma], \text{running}(\bar{F}, \bar{S}) \rangle \\
 \bar{R} = \text{resume}(\bar{F}, \text{None}) \quad Q = \{R \circ \epsilon \mid R \in \bar{R}\} \\
 H_0 = H[o \mapsto \langle \text{Observable}[\sigma], \text{done}(\bar{S}) \rangle] \\
 \forall i \in 1 \dots n. H_i = H_{i-1}[p_i \mapsto \text{unsub}(o, p_i, H)] \\
 \hline
 H, \{ \langle L, x \rangle^{a(o, \bar{p})} \circ FS \} \cup P \rightsquigarrow H_n, \{ FS \} \cup P \cup Q \quad (\text{E-RASYNC-RETURN})
 \end{array}$$

# Example: Terminating an Observable

$$\begin{array}{l}
 H(o) = \langle \text{Observable}[\sigma], \text{running}(\bar{F}, \bar{S}) \rangle \\
 \bar{R} = \text{resume}(\bar{F}, \text{None}) \quad Q = \{R \circ \epsilon \mid R \in \bar{R}\} \\
 H_0 = H[o \mapsto \langle \text{Observable}[\sigma], \text{done}(\bar{S}) \rangle] \\
 \forall i \in 1 \dots n. H_i = H_{i-1}[p_i \mapsto \text{unsub}(o, p_i, H)] \\
 \hline
 H, \{ \langle L, x \rangle^{a(o, \bar{p})} \circ FS \} \cup P \rightsquigarrow H_n, \{ FS \} \cup P \cup Q \quad (\text{E-RASYNC-RETURN})
 \end{array}$$

# Preservation of Heap Evolution

- Proving that reduction preserves heap evolution requires preserving ***non-interference properties***
- Example: a given observable  $o$  can only be waiting for exactly one other observable
- Requires observable ids of waiters to be distinct

# Subject Reduction

## Subject reduction theorem

- ensures ***observable protocol***
- through ***heap evolution*** and ***non-interference***

**Theorem 1** (Subject Reduction). *If  $\vdash H : \star$  and  $\vdash H \text{ ok}$  then:*

1. *If  $H \vdash F : \sigma$ ,  $H \vdash F \text{ ok}$  and  $H, F \longrightarrow H', F'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash F' : \sigma$ ,  $H' \vdash F' \text{ ok}$ , and  $\forall B. H \leq_B H'$ .*
2. *If  $H \vdash FS : \sigma$ ,  $H \vdash FS \text{ ok}$  and  $H, FS \twoheadrightarrow H', FS'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash FS' : \sigma$ ,  $H' \vdash FS' \text{ ok}$  and  $H \leq_{\text{obsIds}(FS)} H'$ .*
3. *If  $H \vdash P : \star$ ,  $H \vdash P \text{ ok}$  and  $H, P \rightsquigarrow H', P'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash P' : \star$  and  $H' \vdash P' \text{ ok}$ .*

# Subject Reduction

## Subject reduction theorem

- ensures **observable protocol**
- through **heap evolution** and **non-interference**

**Theorem 1** (Subject Reduction). *If  $\vdash H : \star$  and  $\vdash H \text{ ok}$  then:*

1. *If  $H \vdash F : \sigma$ ,  $H \vdash F \text{ ok}$  and  $H, F \longrightarrow H', F'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash F' : \sigma$ ,  $H' \vdash F' \text{ ok}$ , and  $\forall B. H \leq_B H'$ .*
2. *If  $H \vdash FS : \sigma$ ,  $H \vdash FS \text{ ok}$  and  $H, FS \twoheadrightarrow H', FS'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash FS' : \sigma$ ,  $H' \vdash FS' \text{ ok}$  and  $H \leq_{\text{obsIds}(FS)} H'$ .*
3. *If  $H \vdash P : \star$ ,  $H \vdash P \text{ ok}$  and  $H, P \rightsquigarrow H', P'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash P' : \star$  and  $H' \vdash P' \text{ ok}$ .*

# Subject Reduction

## Subject reduction theorem

- ensures **observable protocol**
- through **heap evolution** and **non-interference**

**Theorem 1** (Subject Reduction). *If  $\vdash H : \star$  and  $\vdash H \text{ ok}$  then:*

1. *If  $H \vdash F : \sigma$ ,  $H \vdash F \text{ ok}$  and  $H, F \longrightarrow H', F'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash F' : \sigma$ ,  $H' \vdash F' \text{ ok}$ , and  $\forall B. H \leq_B H'$ .*
2. *If  $H \vdash FS : \sigma$ ,  $H \vdash FS \text{ ok}$  and  $H, FS \twoheadrightarrow H', FS'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash FS' : \sigma$ ,  $H' \vdash FS' \text{ ok}$  and  $H \leq_{\text{obsIds}(FS)} H'$ .*
3. *If  $H \vdash P : \star$ ,  $H \vdash P \text{ ok}$  and  $H, P \rightsquigarrow H', P'$  then  $\vdash H' : \star$ ,  $\vdash H' \text{ ok}$ ,  $H' \vdash P' : \star$  and  $H' \vdash P' \text{ ok}$ .*

# Selected Related Work

- Bierman et al. *Pause 'n' Play: Formalizing Asynchronous C#*. ECOOP 2012
- Meijer, Millikin, Bracha. *Spicing Up Dart with Side Effects*. ACM Queue 13.3, 2015
- Meijer. *Your mouse is a database*. CACM 55.5, 2012
- Syme, Petricek, Lomov. *The F# Asynchronous Programming Model*. PADL 2011

# Results

- RAY: unifies Async model and Reactive Extensions
- Operational semantics and static type system
- Proof of subject reduction
  - Based on non-interference properties
  - Ensures observable protocol
- Companion technical report provides details
- See <http://www.csc.kth.se/~phaller/nwpt2015/>

Thank you!

# Results

Questions?

- RAY: unifies Async model and Reactive Extensions
- Operational semantics and static type system
- Proof of subject reduction
  - Based on non-interference properties
  - Ensures observable protocol
- Companion technical report provides details
- See <http://www.csc.kth.se/~phaller/nwpt2015/>