# Techniques and Tools for the Analysis of Timed Workflows

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Joint work with Peter G. Jensen, José A. Mateo and Mathias G. Sørensen.

# Workflow Definition

#### Workflows [Wikipedia]

#### A workflow consists of

- an orchestrated and repeatable pattern of business activity
- enabled by the systematic organization of resources into processes that transform materials, provide services, or process information.

### Examples:

- Car assembly line.
- Insurance claim.
- Blood transfusion.

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All these are examples of time-critical workflows.

There is a need for methods and tools for timed workflow analysis.

- Workflow nets by Wil van der Aalst [ICATPN'97] are widely used for workflow modelling.
- Based on Petri nets.
- Abstraction from data, focus on execution flow.
- Early detection of design errors like deadlocks, livelocks and other abnormal behaviour.
- Classical soundness for workflow nets:
  - option to complete,
  - proper termination, and
  - absence of redundant tasks.

- Theory of workflow nets based on timed-arc Petri nets.
- Definition of soundness and strong soundness.
- Results about decidability/undecidability of soundness.
- Minimum and maximum execution time of workflow nets.
- Integration within the tool TAPAAL and case studies.
- Discrete vs. continuous time.













































## Monotonic Timed-Arc Petri Nets

#### Timed-Arc Petri Nets (TAPN) Modelling Features:

- Timed tokens, intervals (guards) on arcs.
- Weighted arcs.
- Transport arcs.
- Inhibitor arcs.
- Age invariants.
- Urgent transitions.

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#### Monotonic Timed-Arc Petri Nets (MTAPN)

No inhibitor arcs, no age invariants, no urgent transitions.

We consider the integer-delay (discrete-time) semantics (for now).

### Marking in TAPN

$$M: P \rightarrow \mathcal{B}(\mathbb{N}_0)$$

#### Problem

Infinitely many markings even for bounded nets.

We define cut(M) extrapolation for a marking M:

compute for each place maximum relevant token ages

$$C_{max}: P \to (\mathbb{N}_0 \cup \{-1\})$$

• change the age of each token in place p exceeding the bound  $C_{max}(p)$  into  $C_{max}(p) + 1$ .

#### Monotonicity Lemma (t is transition, d is delay)

Let *M* and *M'* be markings in an MTAPN s.t.  $cut(M) \sqsubseteq cut(M')$ .

• If 
$$M \stackrel{t}{\longrightarrow} M_1$$
 then  $M' \stackrel{t}{\longrightarrow} M'_1$  and  $cut(M_1) \sqsubseteq cut(M'_1)$ .

• If 
$$M \stackrel{d}{\longrightarrow} M_1$$
 then  $M' \stackrel{d}{\longrightarrow} M_1'$  and  $cut(M_1) \sqsubseteq cut(M_1')$ .

Fact: inhibitor arcs, age invariants and urgency break monotonicity.

#### Definition

- it has a unique place  $in \in P$  s.t.  $\bullet in = \emptyset$  and  $in^{\bullet} \neq \emptyset$ ,
- it has a unique place  $out \in P$  s.t.  $out^{\bullet} = \emptyset$  and  $\bullet out \neq \emptyset$ ,
- •  $p \neq \emptyset$  and  $p^{\bullet} \neq \emptyset$  for all  $p \in P \setminus \{in, out\}$ , and
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# Timed-Arc Workflow Net

### Definition

A TAPN is called a timed-arc workflow net if

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- **(**) from M it is possible to reach some final marking, and
- 2 if M(out) contains a token then M is a final marking.

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#### Soundness Implies Boundedness

If N is a sound and monotonic timed-arc workflow net then N is bounded.





















Sound and Unbounded Net with Urgent Transitions

Remove age invariant  $\leq 0$  at place  $p_2$  and make  $t_2$  urgent.

Soundness is undecidable for timed-arc workflow nets.

Undecidable even for monotonic nets with only inhibitor arcs, or only age invariants, or only urgent transitions.

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#### Theorem

### Soundness is decidable for

- bounded timed-arc workflow nets, and for
- monotonic timed-arc workflow nets.

Proof: Forward and backward search through the extrapolated state-space (using the function cut). Termination for MTAPN due to the monotonicity lemma.

Notice that for the subclass of monotonic timed-arc Petri nets

- reachability is undecidable [Ruiz, Gomez, Escrig'99], but
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#### Question

Is soundness always sufficient for timed workflows?

### Customer Complaint Workflow



Sound workflow, no timing information, no progress.

### Customer Complaint Workflow



#### Progress is ensured, infinite time-divergent behaviour.

### Customer Complaint Workflow



#### Strongly sound workflow with time-bounded execution.

# Strong Soundness

### Definition

A timed-arc workflow net is strongly sound if

- it is sound,
- has no time-divergent markings (except for the final ones), and
- every infinite computation is time-bounded.

We can define maximum execution time for strongly sound nets.

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We can define maximum execution time for strongly sound nets.

#### Theorem

Strong soundness of timed-arc workflow nets is undecidable.

#### Theorem

Strong soundness of bounded timed-arc workflow nets is decidable.

Proof: By reduction to reachability on timed-arc Petri nets.

# Decidability of Strong Soundness (Proof Sketch)

- Perform normal soundness check and remember the size S of its state-space (in the extrapolated semantics).
- Let B be the maximum possible delay in any marking.
- Check if the given workflow net can delay more than  $U = S \cdot B + 1$  time units before reaching a final marking.
  - If yes, it is not strongly sound.
  - If no, it is strongly sound.



### Implementation and Experiments

- All algorithms implemented within TAPAAL (www.tapaal.net).
- Publicly available and open-source.
- Graphical editor with components, visual simulator.
- Efficient engine implementation (including further optimizations).

Case studies:

- Break System Control Unit, a part of the SAE standard ARP4761 (certification of civil aircrafts).
- MPEG-2 encoding algorithm on multi-core processors.
- Blood transfusion workflow, a larger benchmarking case-study described in little-JIL workflow language.
- Home automation system for light control in a family house with 16 lights/25 buttons, motion sensors and alarm.

# TAPAAL Verification of Break System Control Unit



Simulation Mode: Red transitions are enabled, click a transition to fire it

# TAPAAL Verification of Break System Control Unit



### Recent TAPAAL Development

- TAPAAL is being continuously improved and extended (MPEG-2 workflow analysis with two B-frames took 10s last year, now it takes only 1.4s).
- Memory preserving data structure PTrie.

MPEG-2 with three B-frames

	soundness	strong soundness
no PTrie	33s / 1071MB	30s / 970MB
PTrie	42s / 276MB	45s / 191MB

- Approximate analysis (smaller constants, less precision).
- Compositional, resource-aware analysis.

- Resources with quantitative aspects (cost, energy).
- Two player timed workflow games (also with stochastic opponent).
- Integration with UPPAAL Stratego.
- Workflow analysis in the continuous time semantics.

### Theorem (For Closed TAPNs)

Let  $M_0$  be a marking with integer ages only. If

$$M_0 \stackrel{d_0,t_0}{\longrightarrow} M_1 \stackrel{d_1,t_1}{\longrightarrow} M_2 \stackrel{d_2,t_2}{\longrightarrow} \dots \stackrel{d_{n-1},t_{n-1}}{\longrightarrow} M_n$$

where  $d_i \in \mathbb{R}^{\geq 0}$  then also

$$M_0 \stackrel{d'_0,t_0}{\longrightarrow} M'_1 \stackrel{d'_1,t_1}{\longrightarrow} M'_2 \stackrel{d'_2,t_2}{\longrightarrow} \dots \stackrel{d'_{n-1},t_{n-1}}{\longrightarrow} M'_n$$

where  $d'_i \in \mathbb{N}_0$ .

- We construct a set of linear inequalities that describe all possible delays allowed in the real-time execution.
- We only need difference constraints, hence the corresponding matrix in LP is totally unimodular.
- As the instance of LP has a real solution, it has also an optimal integral solution.

If a timed-arc workflow net is sound in the continuous semantics then it is also sound in the discrete semantics.

Proof:

- Let *N* be sound in the continuous semantics.
- Let M be a marking reachable from the initial marking  $M_{in}$  in the discrete semantics.
- Hence some final marking *M*<sub>out</sub> is reachable from *M* in the continuous semantics.
- We can conclude using the theorem that a marking  $M'_{out}$  with the same distribution of tokens as  $M_{out}$  is reachable from M also in the discrete semantics.

If a timed-arc workflow net with no age invariants and no urgent transitions is sound in the discrete semantics then it is sound also in the continuous semantics.

Proof:

- We can arbitrarily delay in any marking.
- Hence the token ages exceed the maximum constants.
- Now there is no difference between discrete and continuous semantics.

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Proof:

- We can arbitrarily delay in any marking.
- Hence the token ages exceed the maximum constants.
- Now there is no difference between discrete and continuous semantics.

The theorem does not hold for general timed-arc workflow nets.








### Continuous Semantics Challenge



Sound in discrete semantics but unsound in continuous semantics.

- Continuous soundness implies discrete soundness.
- Opposite implication holds only for nets without urgency.
- Strong soundness is not an issue.

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#### Theorem

Let N be a workflow net is sound in the continuous-time semantics.

The net N is strongly sound in the discrete-time semantics iff it is strongly sound in the continuous-time semantics.

# Conclusion

- Framework for the study of timed-arc workflow nets.
- Undecidability of soundness and strong soundness.
- Efficient algorithms for the decidable subclasses.
- Relationship to continuous soundness.
- Integration into the tool TAPAAL.

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#### www.tapaal.net



Silver medal at Model Checking Contest 2014 and 2015. (reachability category)