Using Typings as Types

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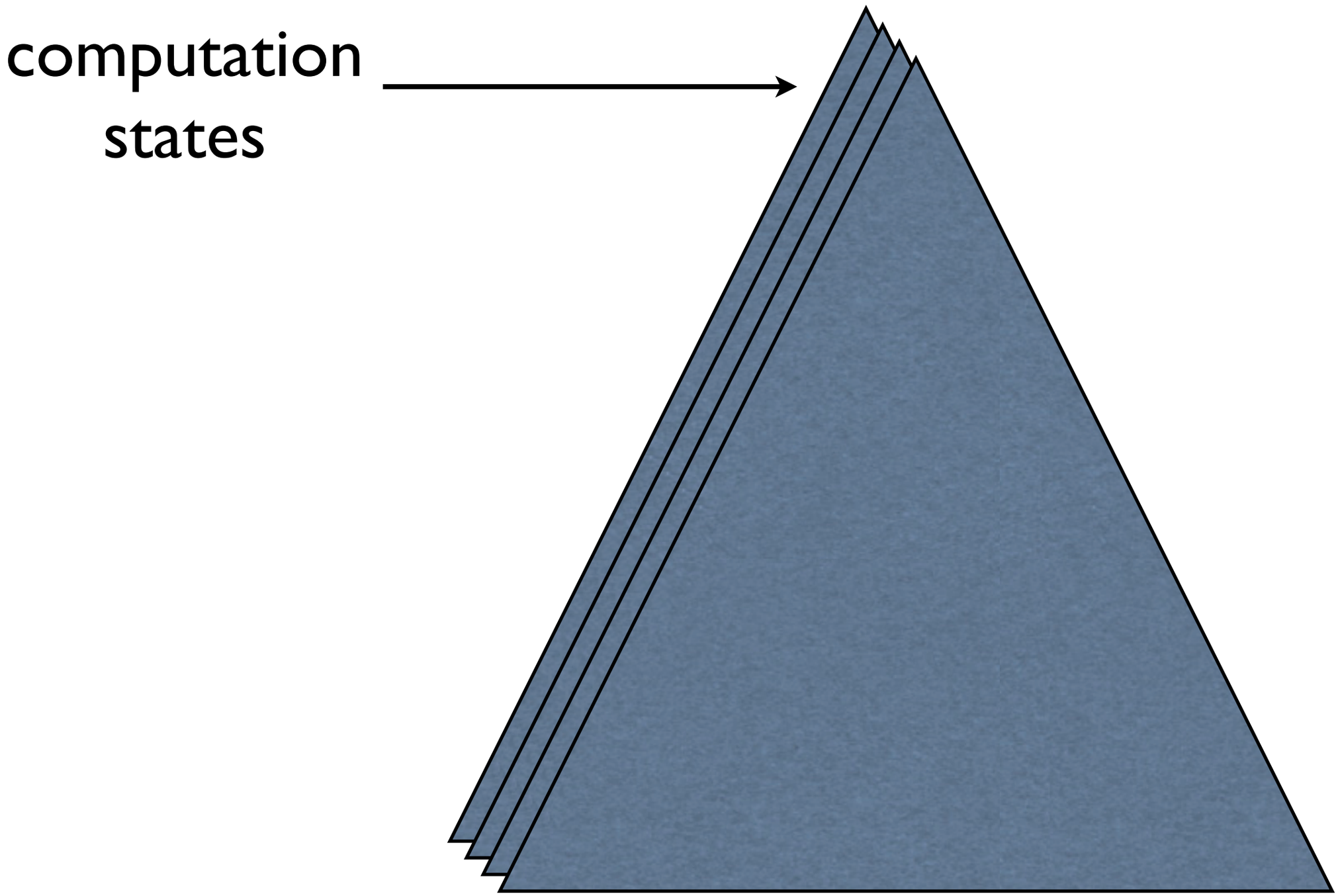
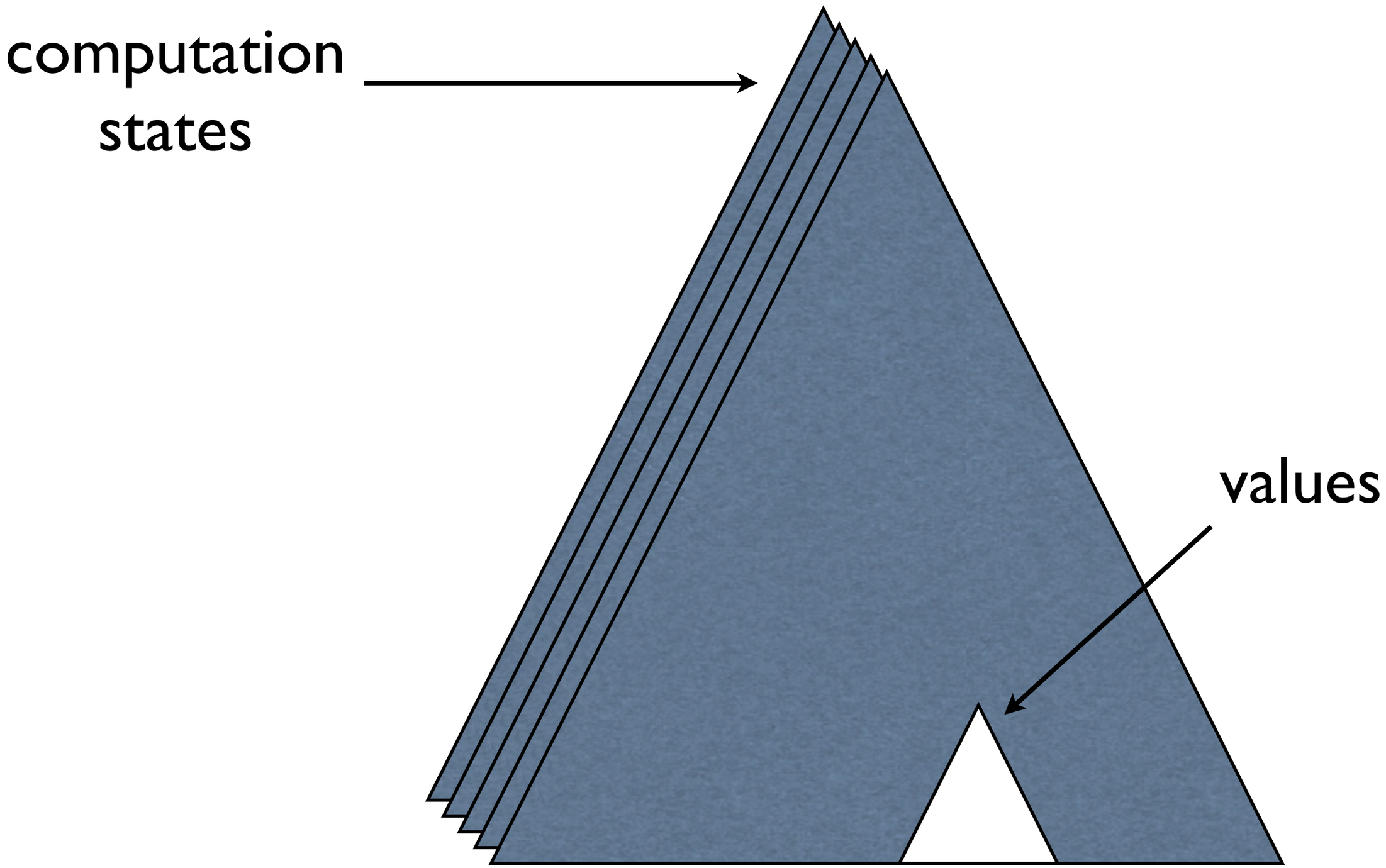
NWPT 2015, Reykjavík, Iceland, October 2015

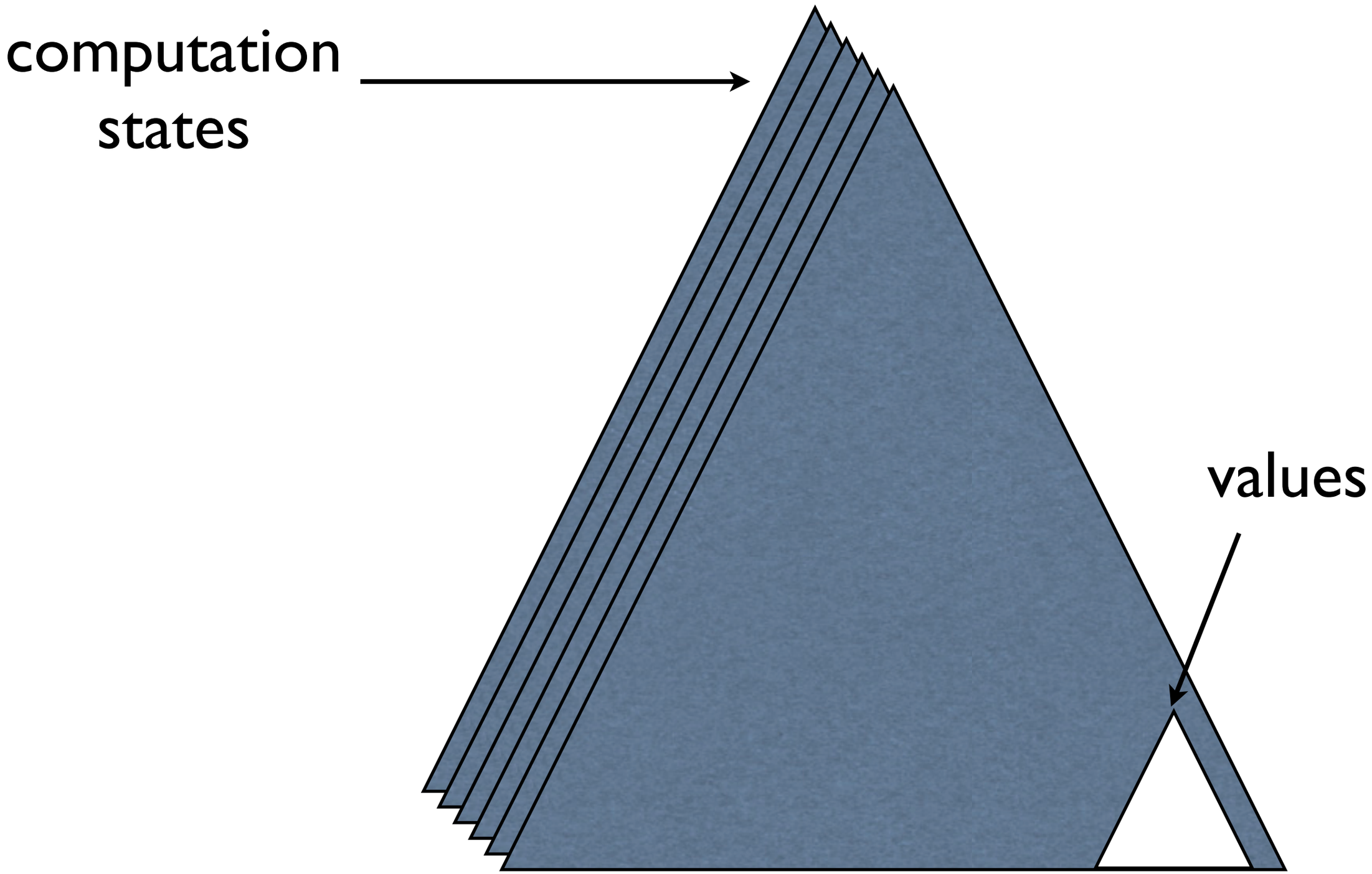
Computations and values



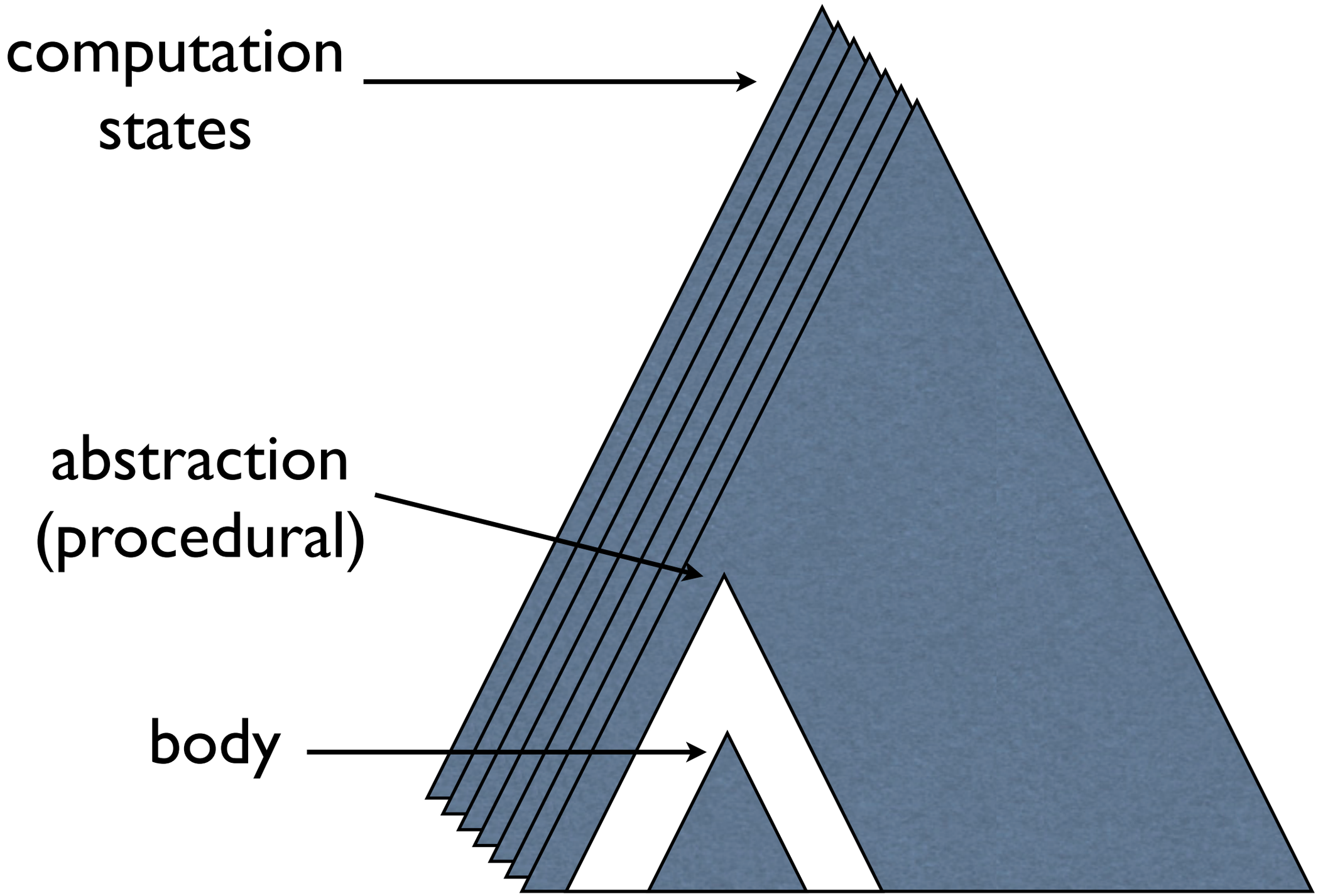
Computations and values



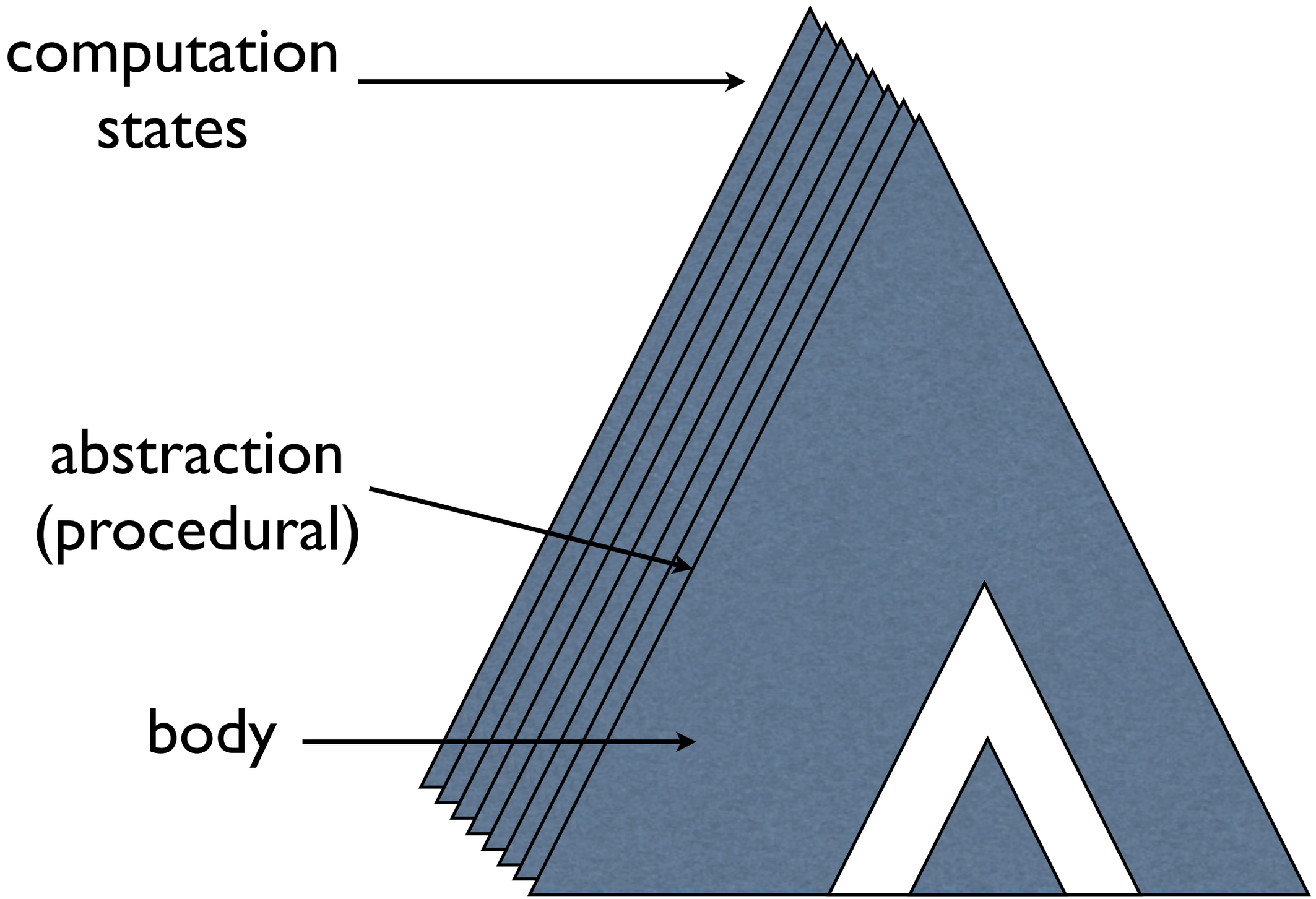
Computations and values



Computations and values



Computations and values



An example of dynamic scope

$$(\lambda f.\ (\lambda y.\ f\ 1)\ 2)\ (\lambda x.\ x + y)$$

$$\text{let } f = (\lambda x.\ x + y) \text{ in let } y = 2 \text{ in } f\ 1$$

$$(\{y : \text{int}\} \vdash (\text{int} \to \text{int})) \qquad \text{int}$$

Static scope

$$M, N ::= n \mid x \mid (M + N) \mid (\lambda x.M) \mid (M\ N)$$

$$\sigma, \tau ::= \mathsf{int} \mid t \mid (\sigma \to \tau)$$

$$\frac{A_x \cup \{x : \sigma\} \vdash M : \tau}{A \vdash (\lambda x.M) : (\sigma \to \tau)} \qquad \frac{A \vdash M : (\sigma \to \tau) \quad A \vdash N : \sigma}{A \vdash (M\ N) : \tau}$$

$$\frac{A \vdash M : \tau}{A' \vdash M : \tau}\ A \subseteq A'$$

Dynamic scope

$$M, N ::= n \mid x \mid (M + N) \mid (\lambda x.M) \mid (M\ N)$$

$$\sigma, \tau ::= \mathsf{int} \mid t \mid (\sigma \to \tau) \mid (A \vdash \tau) \mid (\sigma \wedge \tau)$$

$$\frac{A \vdash M : \tau}{\emptyset \vdash (\lambda x.M) : (A_x \vdash (\sigma \to \tau))}\ (A(x)=\sigma)$$

Dynamic scope

$$M, N ::= n \mid x \mid (M + N) \mid (\lambda x.M) \mid (M\ N)$$

$$\sigma, \tau ::= \mathsf{int} \mid t \mid (\sigma \to \tau) \mid (A \vdash \tau) \mid (\sigma \wedge \tau)$$

$$\frac{A_x \cup \{x : \sigma\} \vdash M : \tau}{\emptyset \vdash (\lambda x.M) : (A_x \vdash (\sigma \to \tau))}$$

$$\frac{A \vdash M : (A' \vdash (\sigma \to \tau)) \quad A \vdash N : \sigma}{(A \wedge A') \vdash (M\ N) : \tau}$$

$$\frac{A \vdash M : \sigma}{A' \vdash M : \tau}\ (A \vdash \sigma) <: (A' \vdash \tau)$$

Explicit closures

$$M ::= \cdots \mid \mathsf{close}(M)$$

$$\frac{A \vdash M : (A' \vdash (\sigma \to \tau))}{(A \wedge A') \vdash \mathsf{close}(M) : (\emptyset \vdash (\sigma \to \tau))}$$

$$\frac{A_x \cup \{x : \sigma\} \vdash M : \tau}{\dfrac{\emptyset \vdash (\lambda x.M) : (A_x \vdash (\sigma \to \tau))}{\emptyset \wedge A_x \vdash \mathsf{close}(\lambda x.M) : (\ \emptyset \vdash (\sigma \to \tau))}}$$

$$\frac{A \vdash M : (\emptyset \vdash (\sigma \to \tau)) \quad A \vdash N : \sigma}{(A \wedge \emptyset) \vdash (M\ N) : \tau}$$

Some subtyping rules

$$\frac{\sigma' <: \sigma \qquad \tau <: \tau'}{(\sigma \to \tau) <: (\sigma' \to \tau')}$$

$$(A_x \cup \{x : \tau\}) <: A_x$$

$$\frac{A' <: A \qquad \sigma <: \tau}{(A \vdash \sigma) <: (A' \vdash \tau)}$$

$$\frac{A_x <: A'_x \qquad \tau <: \tau'}{(A_x \cup \{x : \tau\}) <: (A'_x \cup \{x : \tau'\})}$$

Related work

Implicit Parameters: Dynamic Scoping with Static Types

Jeffrey R. Lewis* Mark B. Shields* Erik Meijer† John Launchbury*

[POPL 2000]

A Polymorphic Modal Type System for Lisp-Like Multi-Staged Languages *

Ik-Soon Kim Kwangkeun Yi Cristiano Calcagno

[POPL 2006]

ISOLATE: A Type System for Self-recursion

Ravi Chugh

[ESOP 2015]

Typings as types

$$A \vdash M : \tau \quad \Rightarrow \quad M : (A \vdash \tau)$$

$$\frac{A_x \cup \{x : \sigma\} \vdash M : \tau}{A \vdash (\lambda x.M) : (\sigma \to \tau)} \quad \Rightarrow \quad \frac{M : (A_x \cup \{x : \sigma\} \vdash \tau)}{(\lambda x.M) : (A \vdash (\sigma \to \tau))}$$

$$\frac{A \vdash M : (\sigma \to \tau) \quad A \vdash N : \sigma}{A \vdash (M\ N) : \tau} \quad \Rightarrow \quad \frac{M : (A \vdash (\sigma \to \tau)) \quad N : (A \vdash \sigma)}{(M\ N) : (A \vdash \tau)}$$

Conclusion

Novel type system

- motivated by considering abstractions as values
- applicable to general dynamic scope
- exploits typings and intersection types

Future work

- meta-theory (soundness, …)
- generalise to include stores, exceptions, …
- achieve modularity
  - *implicit propagation of omitted type features*

Appendix

Implicit parameters ... *[POPL 2000]*

λ-vars	x, y, z		
`let`-vars	p, q		
Implicit vars	$?x, ?y, ?z$		
Terms	t, u, v	$::=$	$x \mid p \mid ?x$
		$\mid$	$\lambda x.\ t \mid t\ u$
		$\mid$	$\texttt{let}\ p = u\ \texttt{in}\ t$
		$\mid$	$t\ \texttt{with}\ ?x = u$
Type vars	α, β		
Types	τ, υ	$::=$	α
		$\mid$	$\upsilon \to \tau$
Schemes	σ, φ	$::=$	$\forall\overline{\alpha}.\ C \Rightarrow \tau$
Contexts	C, D	$::=$	$?x_1 : \tau_1, \ldots, ?x_n : \tau_n$
			$?x_1, \ldots, ?x_n$ distinct
Type contexts	Γ	$::=$	$x_1 : \sigma_1, \ldots, x_n : \sigma_n$
			$x_1, \ldots, x_n$ distinct

Multi-stage ... *[POPL 2006]*

$$\begin{array}{lllll}
\text{Types} & A, B & \in & \mathit{Type} & \\
\text{Type Environments} & \Gamma & \in & \mathit{TyEnv} & = \quad \mathit{Var} \xrightarrow{\mathit{fin}} \mathit{Type} \\
\text{Store Typings} & \Sigma & \in & \mathit{ST} & = \quad \mathit{Loc} \xrightarrow{\mathit{fin}} \mathit{Type}
\end{array}$$

$$\text{Types} \qquad A, B \quad ::= \quad \iota \mid A \to B \mid \Box(\Gamma \triangleright A) \mid A\ \texttt{ref}$$

IsoLate ... self-recursion *[ESOP 2015]*

$$
\begin{array}{rrcl}
\textbf{Types} & R, S, T & ::= & \texttt{unit} \mid \texttt{\{}\ \overline{f}\texttt{:}\overline{T}\ \texttt{\}} \mid S \to T \mid A \mid \mu A.T \mid \forall A.\ T \\
& \textit{pre-type} & \mid & (A : S) \Rightarrow T \\
\textbf{Type Environments} & \Gamma & ::= & - \mid \Gamma, x{:}T \mid \Gamma, A \mid \Gamma, A <: T
\end{array}
$$