Discounted Duration Calculus

Work in Progress

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SCARE
System Correctness
under Adverse Conditions

CARL VON OSSIETZKY universität OLDENBURG

‘Eventually properties’ are common

Booting Eventually the system is ready

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Booting	Eventually the system is ready
Access to shared Resource	After a request, Eventually we get a grant

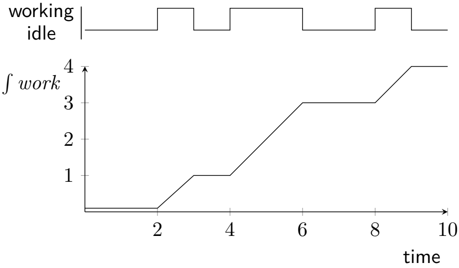
'Eventually properties' are common

Booting	Eventually the system is ready
Access to shared Resource	After a request, Eventually we get a grant

- Often Soon is better than Eventually
- Want quantitative statements about eventually properties
- Truth value is in interval $[0, 1]$

Motivation

Discounting in Duration Caluclus



Example - Energy Consumption

- Working consumes energy
- Idle conserves energy
- Property: The energy lasts long

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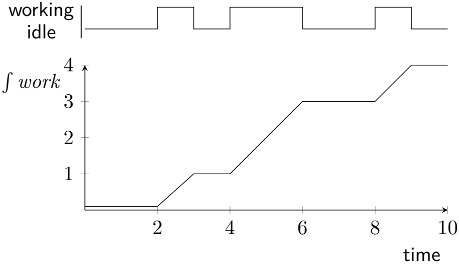
Discounting in Duration Caluclus



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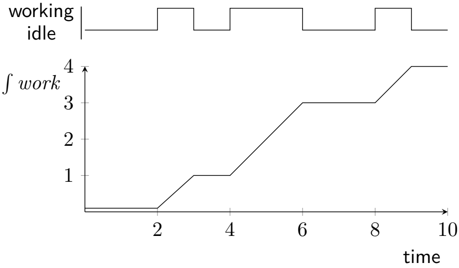
Example - Job Completion

- Job needs a fixed duration of work to finish
- Property: The job is finished soon



Example - Energy Consumption

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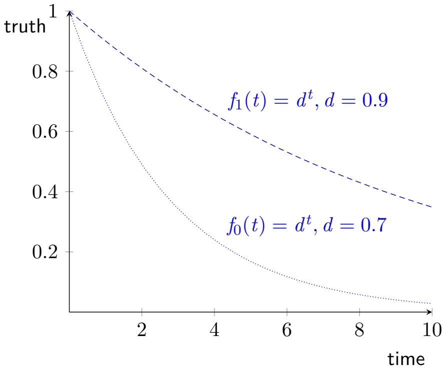
We want properties over durations

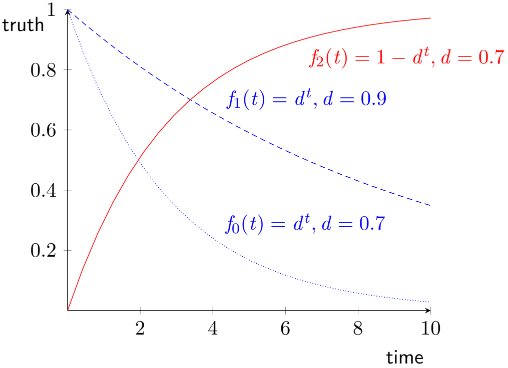
Truth value ...

- ... is a real number from interval $[0, 1]$
- ... represents quality of a system or our satisfaction with the system
- $\mathrm{F}^{d}\phi$ The earlier a good state is reached the better



Truth value ...

- ... is a real number from interval $[0, 1]$
- ... represents quality of a system or our satisfaction with the system
- $\mathrm{F}^d\phi$ The earlier a good state is reached the better
- $\mathrm{G}^d\phi$ The later a bad state is reached the better



Discounting - Background

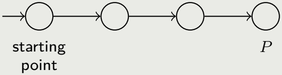
- Introduction in economics [Sam37]
- Introduced into temporal logics in [DAFH$^+$04] (Discounted CTL)
- Definition of discounted LTL [ABK14]

Background and Intuition

Discounting in LTL and CTL

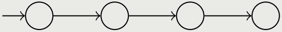
Example

- $\mathrm{F}^{0.7}P$ on path π :



- Discount is the basis
- Time of satisfaction is the exponent

Example

- $\mathrm{F}^{0.7} P$ on path π : 

- Discount is the basis
- Time of satisfaction is the exponent

Truth value is

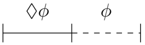
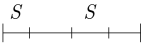
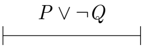
$$\begin{aligned} \mathcal{I}[\![\mathrm{F}^{d} P, \pi]\!] &= \sup_{0 \leq t}\{0.7^{t} \cdot \mathcal{I}[\![P, \pi[t..]]\!]\} \\ &= 0.7^{3} \cdot \mathcal{I}[\![P, \pi[3..]]\!] \\ &= 0.343 \cdot \mathcal{I}[\![P, \pi[3..]]\!] \end{aligned}$$

Discounted Duration Calculus

Syntax

$$\phi ::= \Diamond^d \phi \mid \int^d S \geq c \mid \int^d S > c \mid \neg \phi \mid \phi \vee \phi$$
$$S ::= P \mid \neg S \mid S \wedge S \quad .$$

P is a Boolean proposition

$\Diamond \phi$	There is a neighbouring interval satisfying ϕ	
$\int S \geq c$	State expression S holds for at least c	
	Following abbreviations can be expressed	
$\lceil P \vee \neg Q \rceil$	State expression $P \vee \neg Q$ holds in this interval	
$\Box \phi$	ϕ is satisfied on all neighbouring intervals	

Discounted Duration Calculus

Semantics

$$\mathcal{I}[\![\Diamond^d \phi]\!](tr, [k, m]) = \sup_{l \geq m} \{ d^{l-m} \cdot \mathcal{I}[\![\phi]\!](tr, [m, l]) \}$$

$$\mathcal{I}[\![\textstyle\int S \geq c]\!](tr, [k, m]) = \begin{cases} 0 & \text{if } \int_{t=k}^{m} S(t)\,\mathrm{d}t < c \\ 1 & \text{otherwise} \end{cases}$$

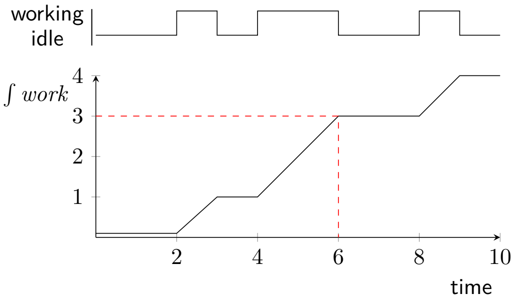
$$\mathcal{I}[\![\neg\phi]\!](tr, [k, m]) = 1 - \mathcal{I}[\![\phi]\!](tr, [k, m])$$

$$\mathcal{I}[\![\phi_0 \vee \phi_1]\!](tr, [k, m]) = \max\{\mathcal{I}[\![\phi_0]\!](tr, [k, m]), \mathcal{I}[\![\phi_1]\!](tr, [k, m])\}$$

Example

Statements over durations

- $\Box^d \int work < 3$



$$\mathcal{I}[\![\Box^d \int work < 3]\!](tr, [k, m])$$
$$= \mathcal{I}[\![\neg\Diamond^d \neg \int work < 3]\!](tr, [k, m])$$

Example

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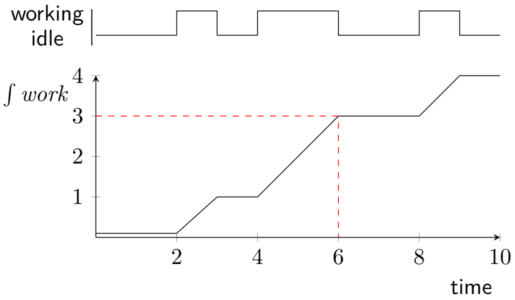

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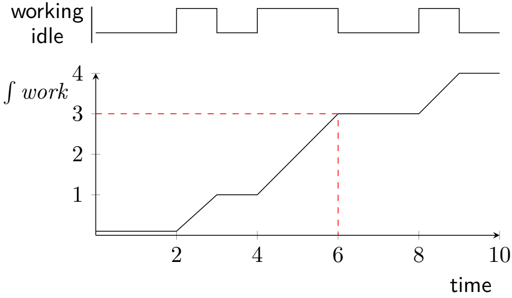
$$= \mathcal{I}[\![\neg \Diamond^d \neg \int work < 3]\!](tr, [k, m])$$
$$= 1 - \sup_{l \geq m}\{d^{l-m} \cdot \mathcal{I}[\![\neg \int work < 3]\!](tr, [m, l])\}$$

Example



Statements over durations

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&= 1 - \sup_{l \geq m}\{d^{l-m} \cdot (1 - \mathcal{I}[\![\textstyle\int work < 3]\!](tr, [m, l]))\}
\end{aligned}
$$

Example

Statements over durations

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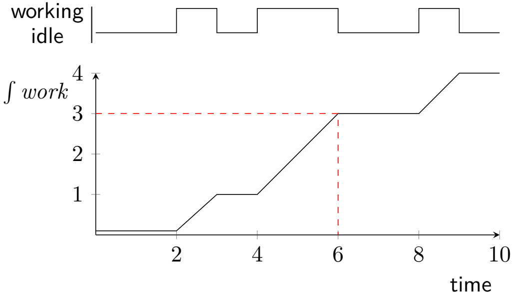

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\end{aligned}$$

with $d = 0.7, m = 0$ $\quad = 1 - 0.7^6 = 0.88$

Questions of Decidability

The formulas we consider

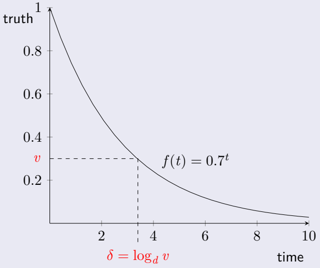
- In negation normal form
- No nested modalities and
- All modalities are discounted

Threshold Satisfiability

Sketch

We can decide $\exists tr.\,\mathcal{I}[\![\phi, tr]\!] \sim v$ with $\sim \in \{<, >, \geq, \leq\}$

- Example $\exists tr.\,\mathcal{I}[\![\phi, tr]\!] \geq v$ with $\phi \equiv \Diamond^{0.7} \int work \geq c$



Truthvalue of $\Diamond^{d}\phi$ when ϕ is satisfied at time t

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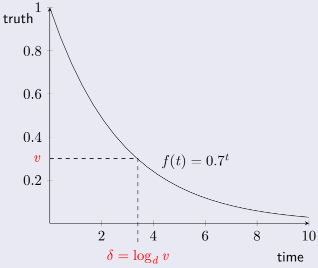
We can decide $\exists tr.\,\mathcal{I}[\![\phi, tr]\!] \sim v$ with $\sim \in \{<, >, \geq, \leq\}$

- Example $\exists tr.\,\mathcal{I}[\![\phi, tr]\!] \geq v$ with $\phi \equiv \Diamond^{0.7} \int work \geq c$
- Transform ϕ into a time-bounded linear hybrid automaton A_ϕ (reachability is decidable) [BDG+11]
- A_ϕ has a location reachable iff $\int work \geq c$ is satisfied in at most δ time



Truthvalue of $\Diamond^d \phi$ when ϕ is satisfied at time t

Example

$\exists tr.\, \mathcal{I}[\![\Diamond^{0.7} \int work \geq c, tr]\!] \geq v$

Threshold Satisfiability

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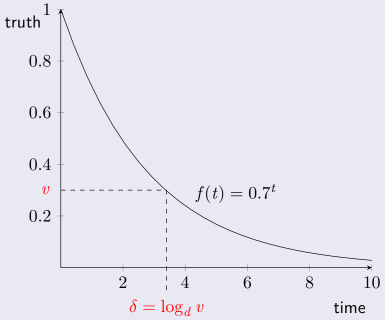

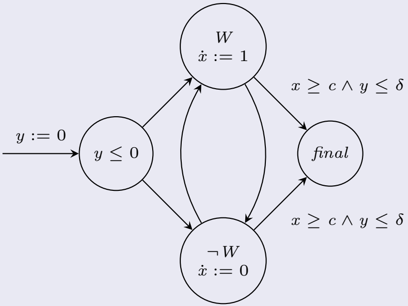

Questions of Decidability

Model Checking — Sketch

Let M be a timed automaton with only clock constraints of the form $x \sim c$, i.e. no comparisons of clocks

- Model checking is $\forall tr \in M . \mathcal{I}[\![\phi, tr]\!] \geq v$
- Equivalently: $\neg \exists tr \in M . \mathcal{I}[\![\phi, tr]\!] < v$
- Check $\exists tr \in M . \mathcal{I}[\![\phi, tr]\!] < v$ on $M \otimes A_\phi$
- A witnessing trace constitutes a counter example

Conclusion

- Gave several examples to show usefulness of our logic
- Some meaningful questions are decidable
- Nested modalities pose a challenge
  I believe I have a procedure for approximate threshold satisfiability

Nested Modalities

Service should be online soon, and then run for a long time $\Diamond^{d_0}\Box^{d_1}\lceil S\rceil$

Future

- Formal proofs of decidability
- Implementation and case studies?

Shaull Almagor, Udi Boker, and Orna Kupferman.
Discounting in LTL.
In *Tools and Algorithms for the Construction and Analysis of Systems*, pages 424–439. Springer, 2014.

Thomas Brihaye, Laurent Doyen, Gilles Geeraerts, Joël Ouaknine, Jean-Fraņcois Raskin, and James Worrell.
On reachability for hybrid automata over bounded time.
In *Automata, Languages and Programming*, pages 416–427. Springer, 2011.

Luca De Alfaro, Marco Faella, Thomas A Henzinger, Rupak Majumdar, and Mariëlle Stoelinga.
Model checking discounted temporal properties.
Springer, 2004.

Paul A Samuelson.
A note on measurement of utility.
The Review of Economic Studies, 4(2):155–161, 1937.

I think we are able to decide approximate threshold satisfiability, i.e. $\exists tr \in M . \mathcal{I}[\![\phi, tr]\!] \sim v \pm \epsilon$

Example

Consider $\Diamond^{d_0} \Box^{d_1} \psi$

- We evaluate the formula at some timepoint δ_i
- The satisfaction value is $d^{\delta_i} \cdot \mathcal{I}[\![\Box^{d_1} \psi, tr]\!]$
- Then the comparison is satisfied iff

$$\mathcal{I}[\![\Box^{d_1} \psi, tr[\delta_i..]]\!] \geq \frac{v}{d^{\delta_i}}$$

- If we pick enough points δ_i at which to try satisfying $\Box^{d_1} \psi$ then we should come within ϵ distance of v, if that is possible at all
- As the discounting formula is exponential hopefully we do not need too many δ_i..?