Discounted Duration Calculus

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1 Introduction

In economics discounting represents that money earned soon can be reinvested earlier and hence yields more revenue than money earned later. Discounting has been introduced into temporal logics to represent that something happening earlier is more important than similar events happening later [DFH$^+$04, ABK14]. A typical example is a rail road crossing. Consider the property 'eventually the gates are open'. While a controller leaving the gates closed an hour after the train has passed might be safe and alive, it is not useful. We can use discounting to express that the controller should not wait unnecessarily long before opening the gates. In [ABK14] this has been described as quantifying the temporal quality of a system.

So far discounting in logics only has been studied for discrete-time temporal logics (LTL, CTL*, μ-calculus) [DFH$^+$04, ABK14]. Here, we study discounting in the dense temporal case. We extend Duration Calculus (DC) [CHR91] with discounting and give some examples on which we want to apply our new logic.

2 Discounted Duration Calculus

We use an adapted version of Duration Calculus (DC), where the chop operator is replaced by left and right neighbourhood modalities. As atomic formulae, we only allow comparison of durations with constants.

Definition 1. *Let $d \in [0,1] \cap \mathbb{Q}, c \in \mathbb{Q}$ and let P be a proposition. Then the syntax of our fragment of DC is defined as*

$$\begin{aligned}\phi &::= \Diamond_{\mathrm{l}}^{d}\phi \mid \Diamond_{\mathrm{r}}^{d}\phi \mid \textstyle\int^{d} S \geq c \mid \int^{d} S > c \mid \neg\phi \mid \phi \vee \phi \mid \lceil S \rceil \enspace, \\ S &::= P \mid \neg S \mid S \wedge S \enspace .\end{aligned}$$

The semantics of DC is defined in terms of trajectories. A trajectory is a function

$$tr : \mathbb{R}_{\geq 0} \to \mathit{Varname} \to \mathbb{B}$$

assigning to each time instant and each variable a truth value.

The semantics does not just determine whether the model satisfies the formula, but rather to what extent the model satisfies the formula. Hence, we associate to a formula and a trajectory

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a *satisfaction value* in the form of a real number in the interval $[0, 1]$. The right neighbourhood modality $\Diamond_{\mathrm{r}}^{d}$ expresses that right of the current interval there is an adjacent interval satisfying the subformula. The discount d is used to reduce the satisfaction value w.r.t. the length of the adjacent interval. The left neighbourhood modality is defined similarly. Discounting in the integral formula represents that it takes at least c units of time to satisfy the comparison. Hence, the first c time units should not be discounted.

Definition 2. *Given a formula, a trajectory tr, and a time interval $[k, m]$, the semantics is defined as*

$$
\begin{aligned}
\mathcal{I}[\![\Diamond_{\mathrm{r}}^{d}\phi]\!](tr,[k,m]) &= \sup_{l \geq m}\{d^{l-m}\cdot \mathcal{I}[\![\phi]\!](tr,[m,l])\}\\
\mathcal{I}[\![\Diamond_{\mathrm{l}}^{d}\phi]\!](tr,[k,m]) &= \sup_{l \leq k}\{d^{k-l}\cdot \mathcal{I}[\![\phi]\!](tr,[l,k])\}\\
\mathcal{I}[\![\textstyle\int^{d} S \geq c]\!](tr,[k,m]) &= \begin{cases} 0 & \text{if } \int_{t=k}^{m} S(t)\,\mathrm{d}t < c\\ d^{m-k-c} & \text{otherwise}\end{cases}\\
\mathcal{I}[\![\textstyle\int^{d} S > c]\!](tr,[k,m]) &= \begin{cases} 0 & \textit{if } \int_{t=k}^{m} S(t)\,\mathrm{d}t \leq c\\ d^{m-k-c} & \text{otherwise}\end{cases}\\
\mathcal{I}[\![\neg\phi]\!](tr,[k,m]) &= 1-\mathcal{I}[\![\phi]\!](tr,[k,m])\\
\mathcal{I}[\![\phi_0\vee\phi_1]\!](tr,[k,m]) &= \max\{\mathcal{I}[\![\phi_0]\!](tr,[k,m]),\mathcal{I}[\![\phi_1]\!](tr,[k,m])\}\\
\mathcal{I}[\![\lceil\phi\rceil]\!](tr,[k,m]) &= \begin{cases} 1 & \text{if } \int_{t=k}^{m} S(t)\,\mathrm{d}t = m-k \text{ and } m>k\\ 0 & \text{otherwise}\end{cases}
\end{aligned}
$$

If we want to use the modalities without discounting we use a discount of 1. In this case we do not explicitly write the discount. Additionally, we define as abbreviations modalities for every right and left adjacent interval and another modality for every interval:

$$\Box_{\mathrm{r}}\phi \overset{\mathrm{def}}{=} \neg\Diamond_{\mathrm{r}}\neg\phi \qquad \Box_{\mathrm{l}}\phi \overset{\mathrm{def}}{=} \neg\Diamond_{\mathrm{l}}\neg\phi \qquad \Box\phi \overset{\mathrm{def}}{=} \Box_{\mathrm{r}}\Box_{\mathrm{r}}\Box_{\mathrm{l}}\Box_{\mathrm{l}}\phi$$

3 Examples

3.1 Call Centre

Consider a customer calling a call centre with a request (see Figure 1). Let S be a state variable indicating whether the customer is waiting or interacting with the employee, let c be the time of interaction between the customer and the employee required to complete the request and let d be the factor of discounting or inflation representing the impatience of the customer. Then a high satisfaction value of the formula

$$\Diamond_{\mathrm{r}}\textstyle\int^{d} S \geq c\ ,$$

by the model of the call centre indicates an efficient handling of requests.

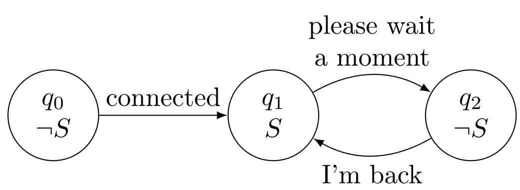

Figure 1: Automaton modelling a call to a call centre. In state q_0 the customer waits to be connected to an employee. In state q_1 the customer interacts with the employee. In state q_2 the employee interacts with his colleagues

3.2 Railway Crossing

Usually at a railway crossing the gates would close when a train is approaching. However, when the street is heavily used the throughput of the street might be too low when the gates are closed whenever a train is approaching. Instead, there could be interlocks for the railway that are nlocked whenever the gates for the street are open. When a train is approaching the gates of the street should however be closed soon. This is expressed as

$$\Box(\lceil apr = 1 \rceil \implies \Diamond_{\mathrm{r}}^{0.9} \lceil SG.closed \rceil) \enspace ,$$

where apr is the number of trains approaching and SG stands for street gate. If another train approaches, e.g. on a parallel track or behind the first train, the urgency is increased, which can be expressed by reducing the discount as in

$$\Box(\lceil apr = 2 \rceil \implies \Diamond_{\mathrm{r}}^{0.8} \lceil SG.closed \rceil) \enspace .$$

3.3 Energy Consumption

Consider a system with an energy capacity c, dissipating energy while operational, but conserving energy in idle mode. Examples are distributed sensors or mobile robots. Let the state variable S describe whether the system currently is operating. Then the satisfaction value of

$$\Box_{\mathrm{r}}^{0.9}(\textstyle\int S \le c)$$

becomes the higher, the longer the system has not used up its energy budget.

4 Outlook

We introduced discounted DC and showed some small examples for which interesting properties can be expressed in our logic. Next we are interested in meaningful decision problems that can be answered automatically. For this we consider threshold satisfaction as in [ABK14], i.e. for a formula ϕ and a trajectory tr, an initial interval $[k, m]$ and $v \in]0, 1[$ we ask

$$\mathcal{I}[\![\phi]\!](tr, [k, m]) \ge v \enspace .$$

For formulas of the form such as in the call centre example the inequality becomes $d^\delta \ge v$, where δ is the length of the interval that satisfies the formula. Then $\delta = \log_d v$ is the maximal δ such that the inequality is still satisfied. Hence, we only have to check a bounded part of the possibly infinite trace. If we additionally assume that the number of state changes in any finite interval is bounded by some constant then threshold satisfaction seems decidable, because there is a bound on the maximal number of state changes we need to consider. For more general formulae, we will investigate approximability of maximal satisfaction values, as facilitated by discounting the far future and past.

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