Logic for Privacy in Security Protocols

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Why Privacy?

Vote in public?

- Advantage: Verifiability
- Serious disadvantage:
  You may not be free to vote what you want.
  - ★ Your boss, spouse, friends, potential future employer can see what you vote.
  - ★ Somebody may bribe or threaten you for voting.

General need for privacy:

- If your actions are observable it can mean subtle restrictions on your freedom.

Alpha-Beta Privacy

Alpha-Beta Privacy

- Novel approach based on Herbrand logic
- Declarative privacy goal specification
  - ★ Specify what private information you deliberately release
  - ★ Allows for incremental approach: discovering the strongest privacy property.
- Reachability problem
  - ★ There is just one reality in each state
- Easier to reason about
  - ★ manually: often easy proof arguments
  - ★ automatically: symbolic/rewriting approaches
  - ★ noname Tool: new automated analysis for bounded sessions
- Deeper understanding: relating to existing approaches

Idea

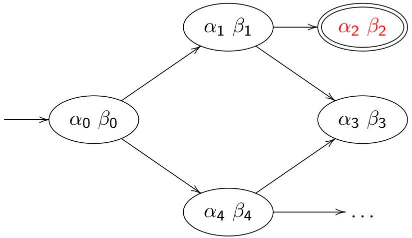
Inspiration/Idea

In zero-knowledge proofs we can usually specify a statement that is being proved.

- Definitely, that statement is revealed to the verifier
  - e.g. "Alice is over 18"
- The verifier (or others) should not learn anything else
  - e.g. "Alice is over 65"
- Everybody can draw conclusions from everything they learned
  - e.g. "Alice is over 15"

Can we do something logical in general for privacy?

State Space



Every state includes two formulae:

- α_i: the information that has been deliberately released so far
  - ★ e.g. the end result of an election
- β_i: the observations that the intruder has made so far.
  - ★ e.g. cryptographic messages exchanged

Attack states:

- when β_i allows the intruder to derive more than α_i.

Alphabet Σ contains:

- cryptographic functions and predicates to represent intruder knowledge
- distinguished subset $\Sigma_0 \subseteq \Sigma$ the high-level information
  - e.g. voters, candidates, natural numbers

In every state:

- α over alphabet Σ_0
- β over alphabet Σ
- $fv(\alpha) \subseteq fv(\beta)$

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- cryptographic functions and predicates to represent intruder knowledge
- distinguished subset $\Sigma_0 \subseteq \Sigma$ the high-level information
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In every state:

- α over alphabet Σ_0
- β over alphabet Σ
- $\mathit{fv}(\alpha) \subseteq \mathit{fv}(\beta)$

Definition (α-β privacy)

Privacy in a state (α, β) holds iff
for every Σ_0-model $\mathcal{I} \models \alpha$ exists a Σ-model $\mathcal{I}' \models \beta$ such that
$\mathcal{I}$ and $\mathcal{I}'$ agree on the interpretation of the symbols in Σ_0 and $\mathit{fv}(\alpha)$.

Thus from β the intruder does not learn anything (except "technical" stuff) that is not implied by α already.

Example

Three RFID tags have interacted with the airport passport reader:

$$\alpha \equiv x_1, x_2, x_3 \in \texttt{Agent}$$

The intruder has observed some messages that allow to deduce

$$\beta \models x_1 \dot{\neq} x_3$$

This violates α-β privacy
because for some models of α there is no corresponding model of β.

Intruder Performs A Symbolic Execution

Example Transaction

$$\star\ x \in \texttt{Agent}.\ \star\ \ y \in \{\mathsf{yes}, \mathsf{no}\}.$$

$$
\begin{array}{l}
\mathsf{rcv}(M).\ \mathsf{try}\ N \doteq \mathsf{dcrypt}(\mathsf{inv}(\mathsf{pk}(\mathsf{s})), M) \\
\qquad \mathsf{in\ if}\ y \doteq \mathsf{yes\ then}\ \nu r.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x), \mathsf{pair}(\mathsf{yes}, N), r)) \\
\qquad\qquad\qquad \mathsf{else}\ \nu r.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x), \mathsf{no}, r)) \\
\qquad \mathsf{catch}\ 0
\end{array}
$$

α	$x \in \texttt{Agent}, y \in \{\mathsf{yes}, \mathsf{no}\}$
β	*true*
γ	$x \doteq a, y \doteq \mathsf{yes}$

The intruder knows that x and y are picked from the respective domains.
γ: what really happened—not seen by intruder.

Intruder Performs A Symbolic Execution

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α	$x \in \mathtt{Agent}, y \in \{\mathsf{yes}, \mathsf{no}\}$
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Intruder can pick any recipe r for M:

- intruder knowledge, closed under public functions
- there infinitely many
- say $r = \mathsf{crypt}(\mathsf{pk}(s), a)$

Intruder Performs A Symbolic Execution

Example Transaction

$$\text{try } N \doteq \text{dcrypt}(\text{inv}(\text{pk}(\text{s})), \text{crypt}(\text{pk}(s), a))$$
$$\text{in if } y \doteq \text{yes then } \nu r.\text{snd}(\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, N), r))$$
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Algebra: $\text{dcrypt}(\text{inv}(x), \text{crypt}(x, y, z)) =_E y$
Thus: decryption works $N \doteq a$—and the intruder knows it.

Intruder Performs A Symbolic Execution

Example Transaction

$$\text{if } y \doteq \text{yes then } \nu r.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x), \mathsf{pair}(\mathsf{yes}, a), r))$$
$$\text{else } \nu r.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x), \mathsf{no}, r))$$

α	$x \in \mathtt{Agent}, y \in \{\text{yes}, \text{no}\}$			
β		$struct_1$ $\phi_1 \equiv y \doteq \text{yes}$	$struct_2$ $\phi_2 \equiv y \doteq \text{no}$	$concr$
	l_1	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{pair}(\mathsf{yes}, a), r)$	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{no}, r)$	$\mathsf{crypt}(\mathsf{pk}(a), \mathsf{pair}(\mathsf{yes}, a), r)$
	$\wedge \bigvee_{i=1}^{2} \phi_i \wedge struct_i \sim concr$			
γ	$x \doteq a, y \doteq \text{yes}$			

The intruder does not know whether the condition is true:

- structural knowledge $struct_1$ or $struct_2$ – the structure the message could have
- *concr* – the concrete message observed.
- one of the ϕ_i is the case and *concr* is statically equivalent to $struct_i$.

Static Equivalence of Frames

$F_1 \sim F_2$ iff for all recipes r_1, r_2:

$$F_1(r_1) \doteq F_1(r_2) \text{ iff } F_2(r_1) \doteq F_2(r_2).$$

Example: encryption without randomization:

	$struct_1$ $\phi_1 \equiv y \doteq \text{yes}$	$struct_2$ $\phi_2 \equiv y \doteq \text{no}$	$concr$
l_1	$\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, a))$	$\text{crypt}(\text{pk}(x), \text{no})$	$\text{crypt}(\text{pk}(a), \text{pair}(\text{yes}, a))$

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- $r_1 = l_1$ and $r_2 = \text{crypt}(\text{pk}(a), no)$ is
  - ★ unequal in *concr*
  - ★ but equal in $struct_2$ if $x \doteq a$.

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- Thus, $\beta \models \neg(x \doteq a \wedge y = \text{no})$ which does not follow from α.

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Example: encryption without randomization:

	struct$_1$ $\phi_1 \equiv y \doteq \mathsf{yes}$	*struct*$_2$ $\phi_2 \equiv y \doteq \mathsf{no}$	*concr*
l_1	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{pair}(\mathsf{yes}, a))$	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{no})$	$\mathsf{crypt}(\mathsf{pk}(a), \mathsf{pair}(\mathsf{yes}, a))$

- $r_1 = l_1$ and $r_2 = \mathsf{crypt}(\mathsf{pk}(a), no)$ is
  - unequal in *concr*
  - but equal in *struct*$_2$ if $x \doteq a$.
- Thus, $\beta \models \neg(x \doteq a \wedge y = \mathsf{no})$ which does not follow from α.
- The same experiment works for any $x \in \mathtt{Agent}$. Thus even $\beta \models y = \mathsf{yes}$.

Another Round

Example Transaction

$\star\ x' \in \mathtt{Agent}.\ \star\ y' \in \{\text{yes}, \text{no}\}.$

$\text{rcv}(M').\ \text{try}\ N' \doteq \text{dcrypt}(\text{inv}(\text{pk}(\text{s})), M')$

$\quad \text{in if}\ y' \doteq \text{yes then}\ \nu r'.\text{snd}(\text{crypt}(\text{pk}(x'), \text{pair}(\text{yes}, N'), r'))$

$\quad\quad \text{else}\ \nu r'.\text{snd}(\text{crypt}(\text{pk}(x'), \text{no}, r'))$

$\quad \text{catch}\ 0$

α	$x \in \mathtt{Agent}, y \in \{\text{yes}, \text{no}\}, x' \in \mathtt{Agent}, y' \in \{yes, no\}$			
β		$struct_1$ $\phi_1 \equiv y \doteq \text{yes}$	$struct_2$ $\phi_2 \equiv y \doteq \text{no}$	$concr$
	l_1	$\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, a), r)$	$\text{crypt}(\text{pk}(x), \text{no}, r)$	$\text{crypt}(\text{pk}(a), \text{pair}(\text{yes}, a), r)$
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γ	$x \doteq a, y \doteq \text{yes}, x' \doteq b, y' \doteq \text{no}$			

Another Round

Example Transaction

$$
\begin{aligned}
\mathsf{rcv}(M').\ & \mathsf{try}\ N' \doteq \mathsf{dcrypt}(\mathsf{inv}(\mathsf{pk}(s)), M') \\
& \mathsf{in\ if}\ y' \doteq \mathsf{yes\ then}\ \nu r'.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x'), \mathsf{pair}(\mathsf{yes}, N'), r')) \\
& \qquad\qquad \mathsf{else}\ \nu r'.\mathsf{snd}(\mathsf{crypt}(\mathsf{pk}(x'), \mathsf{no}, r')) \\
& \mathsf{catch}\ 0
\end{aligned}
$$

α	$x \in \mathtt{Agent}, y \in \{\mathsf{yes}, \mathsf{no}\}, x' \in \mathtt{Agent}, y' \in \{yes, no\}$			
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	l_1	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{pair}(\mathsf{yes}, a), r)$	$\mathsf{crypt}(\mathsf{pk}(x), \mathsf{no}, r)$	$\mathsf{crypt}(\mathsf{pk}(a), \mathsf{pair}(\mathsf{yes}, a), r)$
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γ	$x \doteq a, y \doteq \mathsf{yes}, x' \doteq b, y' \doteq \mathsf{no}$			

Let's use l_1 as input message!

Another Round

Example Transaction

$$\text{try } N' \doteq \text{dcrypt}(\text{inv}(\text{pk}(s)), \text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, a), r))$$
$$\text{in if } y' \doteq \text{yes then } \nu r'.\text{snd}(\text{crypt}(\text{pk}(x'), \text{pair}(\text{yes}, N'), r'))$$
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α	$x \in \texttt{Agent}, y \in \{\text{yes}, \text{no}\}, x' \in \texttt{Agent}, y' \in \{yes, no\}$			
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	l_1	$\text{crypt}(\text{pk}(x), \text{pair}(\text{yes}, a), r)$	$\text{crypt}(\text{pk}(x), \text{no}, r)$	$\text{crypt}(\text{pk}(a), \text{pair}(\text{yes}, a), r)$
	$\wedge \bigvee_{i=0}^{2} \phi_i \wedge struct_i \sim concr$			
γ	$x \doteq a, y \doteq \text{yes}, x' \doteq b, y' \doteq \text{no}$			

Now the intruder cannot tell whether the decryption works—it depends on whether $x \doteq s$.

Another Round

Evaluating the conditions gives now 6 cases:

$$
\begin{array}{lll|ll}
x \doteq s & y' \doteq \mathsf{yes} & y \doteq \mathsf{yes} & \mathsf{snd}(\ldots yes) & struct_1 \\
x \doteq s & y' \doteq \mathsf{yes} & y \doteq \mathsf{no} & \mathsf{snd}(\ldots yes) & struct_2 \\
x \doteq s & y' \doteq \mathsf{no} & y \doteq \mathsf{yes} & \mathsf{snd}(\ldots no) & struct_1 \\
x \doteq s & y' \doteq \mathsf{no} & y \doteq \mathsf{no} & \mathsf{snd}(\ldots no) & struct_2 \\
x \not\doteq s & & y \doteq \mathsf{yes} & 0 & struct_1 \\
x \not\doteq s & & y \doteq \mathsf{no} & 0 & struct_2
\end{array}
$$

Another Round

Since the intruder can observe that no message is sent, only two cases remain:

$$\begin{array}{ccc|cc} x \dot{\neq} s & & y \doteq \mathsf{yes} & 0 & struct_1 \\ x \dot{\neq} s & & y \doteq \mathsf{no} & 0 & struct_2 \end{array}$$

Thus the intruder can derive: $\beta \models x \dot{\neq} s$.

Strongest Privacy Goal

In general, when detecting such a violation of (α, β)-privacy, one has two options:

- Strengthen the protocol, e.g., send a decoy message instead of 0.
- Declassification of some information, e.g., release to α that $x \dot{\neq} s$.

Incremental exploration of the strongest privacy goal that a protocol can achieve

- Start with no α-releases (just domain constraints).
- Whenever a violation is found, make a minimal release that fixes that violation.
- Repeat until no more violations are found.

Examples:

- Abadi-Fournet protocol from *Private Authentication*, TCS 2004.
- ICAO BAC – e.g. French vs. British implementation

Noname Tool

A decision procedure for (α, β)-privacy for a bounded number of transitions.

- Symbolic representation for the non-deterministic choices
- Symbolic representation for intruder-chosen recipes
- Handling of constructor/destructor theories
- Number of Case Studies (Unlinkability, Privacy)

Ask for more on Noname and attacks :-)

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